Lecture 12 – Probability

DSC 10, Fall 2021

Announcements

- Lab 4 due **tomorrow at 11:59pm**.
- Homework 4 due **Saturday 10/23 at 11:59pm**.
- Project 1 due **Tuesday 11/2 at 11:59pm**.
    - Pushed back!
- Midterm Exam **next Wednesday 10/27 during lecture**.
- Remote discussion next week is converted to a Midterm Review Session; remote discussion after that is cancelled.

Midterm Exam details

- Midterm Exam **next Wednesday 10/27 during lecture**.
    - You **must** take it during your assigned lecture slot.
- It will be a 50 minute exam, administered through Gradescope.
    - e.g. If you are in Lecture C00, it will appear on Gradescope at exactly 9:00AM and you will have to submit it at exactly 9:50AM.
    - **DO NOT OPEN THE EXAMS FOR OTHER SECTIONS**! If you do so, you will get a 0.
- Format: mix of multiple choice and short answer.
    - You will write all of your code directly on Gradescope, **not** in a Jupyter Notebook.
- Rules: Open notes, open internet, must work alone.
    - You're not allowed to talk to **anyone** during the exam, in-person or online, even if they have nothing to do with DSC 10.
    - You also should not discuss the exam with anyone after you take it until we make a Campuswire post saying all exams are over.
    - No posting on Campuswire, unless it is to ask us a question via private post.
- More details to come.

Agenda

- Review and finish iteration from last lecture.
- Introduction to probability.

Iteration

In [ ]:

```
teams = ["bunnies", "colts", "ducklings", "fawns", "joeys",
         "lambs", "piglets", "porcupettes", "puppies", "tadpoles"]

for team in teams:
    print('Go ' + team + '!!!')
```

Building an array by iterating

- **Question: How many letters are in each team's name?**
- We can figure it out one team at a time, but we want to save our results!
- To do this, we use `np.append`, which appends (adds) an element to the end of an array.
- We will follow this pattern **very often** when generating data and running experiments or simulations.

In [ ]:

```
teams
```

In [ ]:

```
# Creating an empty array to store our results
lengths = np.array([])

for team in teams:
    lengths = np.append(lengths, len(team))

lengths
```

Working with strings

- String are sequences, so we can iterate over them, too!

In [ ]:
```
for letter in 'uc san diego':
    print(letter.upper())
```

In [ ]:
```
'california'.count('a')
```

Discussion Question

Fill in the two blanks so that `vowel_count` returns the number of vowels in the input string `s`.

```
def vowel_count(s):
    number = 0
    for vowel in ______:
        number = number + ______
    return number
```

A. `s`; `'aeiou'.count(vowel)`

B. `vowels`; `s.count(vowel)`

C. `'aeiou'`; `s[vowel]`

D. `'aeiou'`; `s.count(vowel)`

E. `np.arange(len(vowels))`; `vowel`

To answer, go to [menti.com (https://menti.com)](https://menti.com) and enter the code 5779 8128.

Motivation for probability

Swain vs. Alabama, 1965

- Robert Swain was a black man convicted of crime in Talladega County, Alabama.
- He appealed the jury's decision all the way to the Supreme Court, on the grounds that Talladega County systematically excluded Black people from juries.
- At the time, only men 21 years or older were allowed to serve on juries. 26% of this population was Black.
- But of the 100 men on Robert Swain's jury panel, only 8 were Black.

Supreme Court Ruling

- About disparities between the percentages in the eligible population and the jury panel, the Supreme Court wrote:

"... the overall percentage disparity has been small...”

- The Supreme Court denied Robert Swain’s appeal and he was sentenced.
- The fact that the jury panel had far fewer Black men proportionally than Talladega County is an example of racial bias.
- Over the next few weeks, we will give you tools to quantitatively highlight this bias.
    - We will try to answer the question, "what are the chances that this disparity was due to random chance?"
    - If this chance is small, we know something is wrong.
    - But first: we need to formalize what **probability** is.

Probability theory

Probability theory

- Some things in life *seem* random.
  - e.g. flipping a coin 🪙 or rolling a die 🎲.
- The **probability** of seeing "heads" is 50%.
- One interpretation of probability says that if we flipped a coin infinitely many times, then 50% of outcomes would be heads.

Terminology

- **Experiment**: A process or action whose result is random.
    - e.g., rolling a die.
    - e.g., flipping a coin twice.
- **Outcome**: The result of an experiment.
    - e.g., the possible outcomes of rolling a six-sided die are 1, 2, 3, 4, 5, 6.
    - e.g., the possible outcomes of flipping a coin twice are HH, HT, TH, TT.
- **Event**: A set of outcomes.
    - e.g., the event that the die lands on a even number is the collection of outcomes {2, 4, 6}.
    - e.g., the event that the die lands on a 5 is the collection of outcomes {5}.
    - e.g., the event that there was at least one head in two flips is the collection of outcomes {HH, HT, TH}.

Terminology

- **Probability**: A number between 0 and 1 which describes the likelihood of an event.
    - 0: the event never happens.
    - 1: the event always happens.
- Notation: if A is an event, $P(A)$ is the probability of that event.

Equally-likely outcomes

- If all outcomes in event A are equally likely, then the probability of A is computed as follows:

$$P(A) = \frac{\text{\# of outcomes that make } A \text{ happen}}{\text{total \# of outcomes}}$$

Discussion Question

I have three cards: red, blue, and green. What is the chance that I choose a card at random, and it is green, then – **without putting it back** – I choose another card at random and it is red?

- A) 1/9
- B) 1/6
- C) 1/3
- D) 2/3
- E) None of the above.

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Discussion Question solved

- The possible outcomes are: RG, RB, GR, GB, BR, BG.
- These outcomes are equally-likely.
- There is only one outcome which makes the event happen: GR.
- Hence the probability is $1/6$.

Conditional probabilities

- Two events A and B can both happen.
    - e.g. A is the event "roll is 3 or less", B is the event "roll is even".
- Suppose that we know A has happened, but we don't know if B has.
- The conditional probability of B given A is:

$$P(B \text{ given } A) = \frac{\text{\# of outcomes satisfying both } A \text{ and } B}{\text{\# of outcomes satisfying } A}$$

Discussion Question

$$P(B \text{ given } A) = \frac{\text{\# of outcomes satisfying both } A \text{ and } B}{\text{\# of outcomes satisfying } A}$$

I roll a six-sided die and don't tell you what the result is, but I tell you that it is 3 or less. What is the probability that the result is even?

- A) 1/2
- B) 1/3
- C) 1/4
- D) None of the above.

To answer, go to [menti.com (https://menti.com)](https://menti.com) and enter the code 5779 8128.

Discussion Question solved

$$P(B \text{ given } A) = \frac{\#\text{ of outcomes satisfying both } A \text{ and } B}{\#\text{ of outcomes satisfying } A}$$

I roll a six-sided die and don't tell you what the result is, but I tell you that it is 3 or less. What is the probability that the result is even?

- There are three outcomes where the roll is 3 or less: 1, 2, 3.
- There is only one outcome where both A and B happen: 2.
- So $P(B \text{ given } A) = 1/3$.

Probability that two events both happen

$$P(A \text{ and } B) = \frac{\text{\# of outcomes satisfying both } A \text{ and } B}{\text{total \# of outcomes}}$$

What is the probability that the roll 3 or less and even?

- Only 1 outcome satisfies both: rolling a 2.
- There are 6 total outcomes.
- Thus, $P(A \text{ and } B) = 1/6$.

Another way to compute the probability that two events both happen

$$P(A \text{ and } B) = P(B \text{ given } A) \cdot P(A)$$

What is the probability that the roll 3 or less and even?

- We saw probability of the roll being even given that the roll is 3 or less is 1/3.
- The probability that the roll is 3 or less is 1/2.
- Thus, the probability the roll is both even and 3 or less is $1/3 \cdot 1/2 = 1/6$.
- Note that an equivalent formula is $P(A \text{ and } B) = P(A \text{ given } B) \cdot P(B)$.

What if A isn’t affected by B? 🤔

- We have found that $P(A \text{ and } B) = P(B \text{ given } A) \cdot P(A)$.
- Sometimes $P(B \text{ given } A) = P(B)$. Then $P(A \text{ and } B) = P(A) \cdot P(B)$.
- Example: Suppose we flip a fair coin three times.
    - The probability that the second flip is heads doesn't depend on the result of the first flip.
- What is the probability of getting tails three times in a row?
    - $1/2 \cdot 1/2 \cdot 1/2 = 1/8$.

Probability of either of two events happening

$$P(A \text{ or } B) = \frac{\text{\# of outcomes satisfying either } A \text{ or } B}{\text{total \# of outcomes}}$$

Mutual exclusivity

- Suppose that if A happens, then B doesn't, and if B happens then A doesn't.
- Then the # of outcomes satisfying either A or B is just:

$$(\text{\# of outcomes satisfying } A) + (\text{\# of outcomes satisfying } B)$$

- So **if** A and B are mutually exclusive:

$$P(A \text{ or } B) = \frac{\text{\# of outcomes satisfying either } A \text{ or } B}{\text{total \# of outcomes}}$$

$$= \frac{(\text{\# of outcomes satisfying } A) + (\text{\# of outcomes satisfying } B)}{\text{total \# of outcomes}}$$

$$= \frac{(\text{\# of outcomes satisfying } A)}{\text{total \# of outcomes}} + \frac{(\text{\# of outcomes satisfying } B)}{\text{total \# of outcomes}}$$

$$= P(A) + P(B)$$

Probability that an event *doesn’t* happen

- The probability that A doesn't happen is just $1 - P(A)$.
- Example: If the probability it is sunny tomorrow is 0.85, then the probability it is not sunny tomorrow is 0.15.

Discussion Question

Every time I call my grandma 👵, the probability that she answers her phone is 1/3. If I call my grandma three times today, what is the chance that I will talk to her at least once?

- A) 1/3
- B) 2/3
- C) 1/2
- D) 1
- E) None of the above.

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Discussion Question solved

- Let's first calculate the probability that she doesn't answer her phone in three tries.
    - The probability she doesn't answer her phone on any one attempt is 2/3.
    - So the probability she doesn't answer her phone in three tries is $2/3 \cdot 2/3 \cdot 2/3 = 8/27$.
- But we want the probability of her answering **at least** once. So we subtract the above result from 1.
    - $1 - 8/27 = 19/27$; none of the above!

Summary

Summary

- Probabilities describe the likelihood of an event occurring.
- There are several rules for computing probabilities. We looked at a few special cases:
    - When all outcomes are equally likely.
    - When we know that event A has occurred and want the probability that event B will occur.
    - When we want the probability that both events A and B occur.
    - When we want the probability that event A or event B occurs.
        - When two events are mutually exclusive, they have no shared outcomes.
    - When we want the probability that event A doesn't occur.
- **Next time:** simulations.