Lecture 27 – Final Review

DSC 10, Spring 2024

Announcements

- Lab 7 is due **tomorrow at 11:59PM**.
- The Final Exam is **this Saturday, June 8th from 7-10PM**.
  - You will take the exam in Solis 104 or Solis 107. Seating assignments coming soon..
- If at least 75% of the class fills out both **SETs** and the DSC 10-specific **End-of-Quarter Survey**, then the entire class will have **1% of extra credit added to their overall grade**. We value your feedback!

Agenda

- We'll work through selected problems from the Fall 2023 Final Exam.
- We won't write any code, since you can't run code during the exam. Instead, we'll try to think like the computer ourselves.
- These annotated slides will be posted after lecture is over, as will the solutions to the entire exam.
- **Try the problems with us!**

Fall 2023 Final Exam

Access the exam **here**. Make sure to read the data info sheet at the top before starting.

Problem 6

Aaron wants to explore the discrepancy in fraud rates between `"discover"` transactions and `"mastercard"` transactions. To do so, he creates the DataFrame `ds_mc`, which only contains the rows in `txn` corresponding to `"mastercard"` or `"discover"` transactions.

After he creates `ds_mc`, Aaron groups `ds_mc` on the `"card"` column using two different aggregation methods. The relevant columns in the resulting DataFrames are shown below.

`ds_mc.groupby("card").sum()`

card	is_fraud
discover	160
mastercard	4000

`ds_mc.groupby("card").count()`

card	is_fraud
discover	2000
mastercard	40000

Aaron decides to perform a test of the following pair of hypotheses:

- **Null Hypothesis**: The proportion of fraudulent `"mastercard"` transactions is **the same as** the proportion of fraudulent `"discover"` transactions.
- **Alternative Hypothesis**: The proportion of fraudulent `"mastercard"` transactions is **less than** the proportion of fraudulent `"discover"` transactions.

As his test statistic, Aaron chooses the **difference in proportion of transactions that are fraudulent**, in the order `"mastercard"` minus `"discover"`.

Problem 6

Problem 6.1

What type of statistical test is Aaron performing?

- Standard hypothesis test
- Permutation test

Problem 6.2

What is the value of the observed statistic? Give your answer either as an exact decimal or simplified fraction.

`ds_mc.groupby("card").sum()`

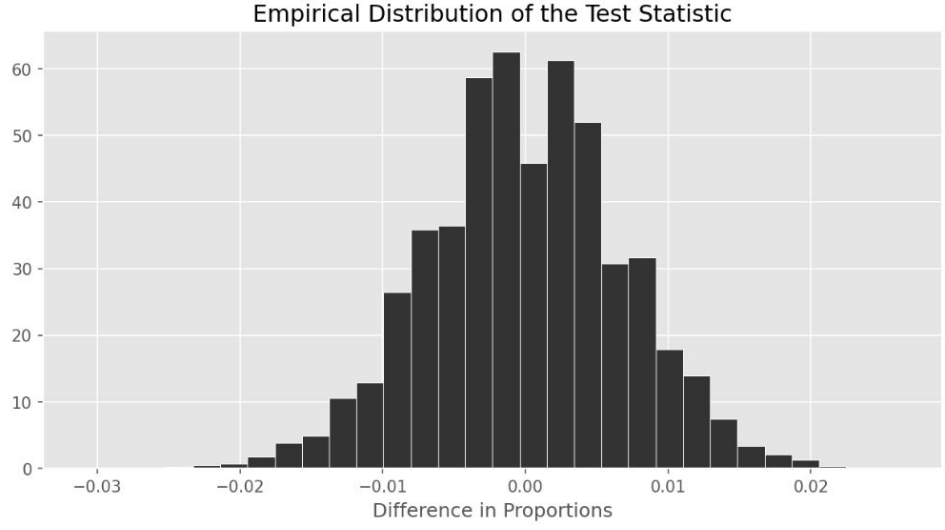
card	is_fraud
discover	160
mastercard	4000

`ds_mc.groupby("card").count()`

card	is_fraud
discover	2000
mastercard	40000

Problem 6

The empirical distribution of Aaron's chosen test statistic is shown below.



Problem 6.3

Which of the following is closest to the p-value of Aaron's test?

- 0.001
- 0.37
- 0.63
- 0.94
- 0.999

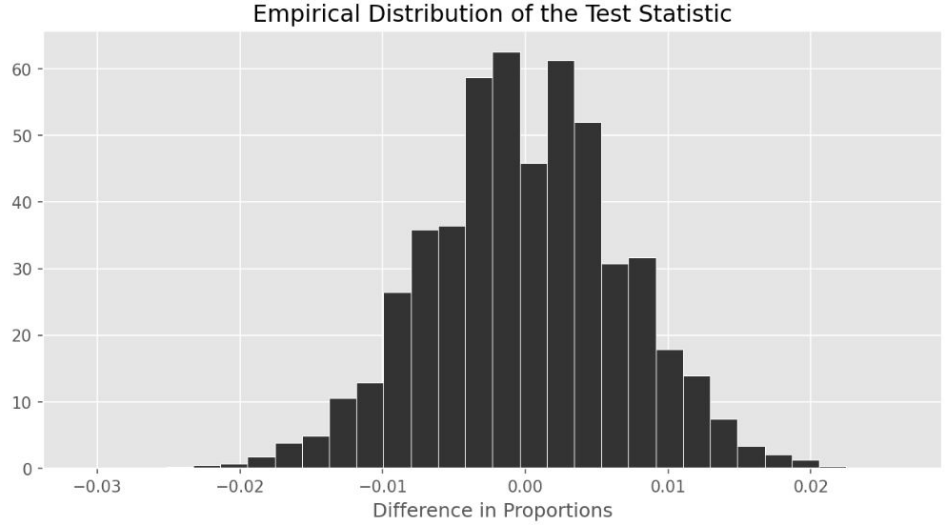
Problem 6.4

What is the conclusion of Aaron's test?

- The proportion of fraudulent `"mastercard"` transactions is **less than** the proportion of fraudulent `"discover"` transactions.
- The proportion of fraudulent `"mastercard"` transactions is **greater than** the proportion of fraudulent `"discover"` transactions.
- The test results are inconclusive.
- None of the above.

Problem 6

The empirical distribution of Aaron's chosen test statistic is shown below.



Problem 6.5

Which of the following is closest to the p-value of Aaron's new test?

- 0.001
- 0.06
- 0.37
- 0.63
- 0.94
- 0.999

Aaron now decides to test a slightly different pair of hypotheses.

- **Null Hypothesis**: The proportion of fraudulent `"mastercard"` transactions is **the same as** the proportion of fraudulent `"discover"` transactions.
- **Alternative Hypothesis**: The proportion of fraudulent `"mastercard"` transactions is **greater than** the proportion of fraudulent `"discover"` transactions.

He uses the same test statistic as before.

Problem 9

The DataFrame `ten_txns`, **displayed in its entirety below**, contains a simple random sample of 10 rows from `txn`.

transaction_id	is_fraud	amount	method	card	lifetime	browser
3169166	True	100.00	credit	visa	532601.00	chrome 63.0
3093921	False	100.00	debit	mastercard	173276.00	mobile safari 10.0
3137058	False	100.00	debit	visa	120000.00	chrome 63.0
3063164	False	100.00	debit	visa	141342.00	mobile safari 11.0
3051461	False	75.00	credit	visa	153871.00	ie 11.0 for desktop
3171154	False	25.00	debit	visa	182654.00	mobile safari generic
3222420	False	25.00	credit	visa	50199.00	safari generic
3226397	False	22.96	debit	mastercard	122352.00	safari generic
3073572	False	9.33	credit	mastercard	66703.00	mobile safari 11.0
3253371	False	5.00	debit	visa	3007.00	chrome 64.0

Problem 9.1

Suppose we randomly select one transaction from `ten_txns`. What is the probability that the selected transaction is made with a `"card"` of `"mastercard"` or a `"method"` of `"debit"`?

Problem 9

The DataFrame `ten_txns`, **displayed in its entirety below**, contains a simple random sample of 10 rows from `txn`.

transaction_id	is_fraud	amount	method	card	lifetime	browser
3169166	True	100.00	credit	visa	532601.00	chrome 63.0
3093921	False	100.00	debit	mastercard	173276.00	mobile safari 10.0
3137058	False	100.00	debit	visa	120000.00	chrome 63.0
3063164	False	100.00	debit	visa	141342.00	mobile safari 11.0
3051461	False	75.00	credit	visa	153871.00	ie 11.0 for desktop
3171154	False	25.00	debit	visa	182654.00	mobile safari generic
3222420	False	25.00	credit	visa	50199.00	safari generic
3226397	False	22.96	debit	mastercard	122352.00	safari generic
3073572	False	9.33	credit	mastercard	66703.00	mobile safari 11.0
3253371	False	5.00	debit	visa	3007.00	chrome 64.0

Problem 9.2

Suppose we randomly select two transactions from `"ten_txns"`, without replacement, and learn that neither of the selected transactions is for an amount of 100 dollars. Given this information, what is the probability that:

- the first transaction is made with a `"card"` of `"visa"` and a `"method"` of `"debit"`, and
- the second transaction is made with a `"card"` of `"visa"` and a `"method"` of `"credit"`?

Problem 11

On Reddit, Yutian read that 22% of all online transactions are fraudulent. She decides to test the following hypotheses:

- **Null Hypothesis**: The proportion of online transactions that are fraudulent is **0.22**.
- **Alternative Hypothesis**: The proportion of online transactions that are fraudulent is not **0.22**.

To test her hypotheses, she decides to create a **95%** confidence interval for the proportion of online transactions that are fraudulent using the Central Limit Theorem.

Unfortunately, she doesn't have access to the entire `txn` DataFrame; rather, she has access to a simple random sample of `txn` of size n. In her sample, the proportion of transactions that are fraudulent is **0.2** (or equivalently, $\frac{1}{5}$).

Problem 11.1

The width of Yutian's confidence interval is of the form

$$\frac{c}{5\sqrt{n}}$$

where n is the size of her sample and c is some positive integer.
What is the value of c? Give your answer as an integer.

Hint: Use the fact that in a collection of 0s and 1s, if the proportion of values that are 1 is p, the standard deviation of the collection is $\sqrt{p(1-p)}$.

Problem 11

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Problem 11.2

There is a positive integer J such that:

- If $n < J$, Yutian will fail to reject her null hypothesis at the **0.05** significance level.
- If $n > J$, Yutian will reject her null hypothesis at the **0.05** significance level.

What is the value of J? Give your answer as an integer.

Problem 12

On Reddit, Keenan also read that 22% of all online transactions are fraudulent. He decides to test the following hypotheses at the **0.16 significance level**:

- **Null Hypothesis**: The proportion of online transactions that are fraudulent is **0.22**.
- **Alternative Hypothesis**: The proportion of online transactions that are fraudulent is not **0.22**.

Keenan has access to a simple random sample of `txn` of size **500**. In his sample, the proportion of transactions that are fraudulent is **0.23**.

Below is an incomplete implementation of the function `reject_null`, which creates a bootstrap-based confidence interval and returns **True** if the conclusion of Keenan's test is to **reject** the null hypothesis, and **False** if the conclusion is to **fail to reject** the null hypothesis, all at the **0.16** significance level.

```
def reject_null():
    fraud_counts = np.array([])
    for i in np.arange(10000):
        fraud_count = np.random.multinomial(500, __(a)__)[0]
        fraud_counts = np.append(fraud_counts, fraud_count)

    L = np.percentile(fraud_counts, __(b)__)
    R = np.percentile(fraud_counts, __(c)__)

    if __(d)__ < L or __(d)__ > R:
        # Return True if we REJECT the null.
        return True
    else:
        # Return False if we FAIL to reject the null.
        return False
```

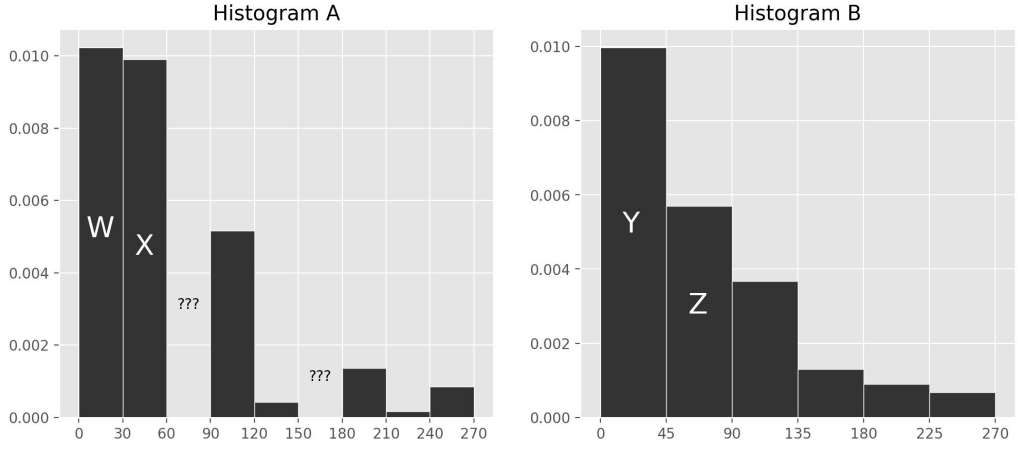
Fill in the blanks so that `reject_null` works as intended.

Hint: Your answer to (d) should be an integer greater than 50.

Problem 13

Ashley doesn't have access to the entire `txn` DataFrame; instead, she has access to a simple random sample of **400** rows of `txn`.

She draws two histograms, each of which depicts the distribution of the `"amount"` column in her sample, using different bins.



Unfortunately, DataHub is being finicky and so two of the bars in Histogram A are deleted after it is created.

Problem 13.1

In Histogram A, which of the following bins contains approximately 60 transactions?

- $[30, 60)$
- $[90, 120)$
- $[120, 150)$
- $[180, 210)$

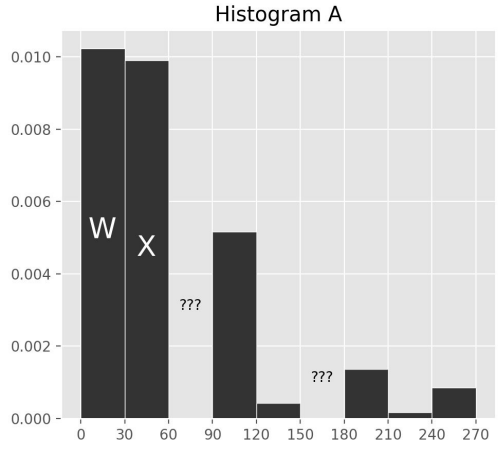
Problem 13

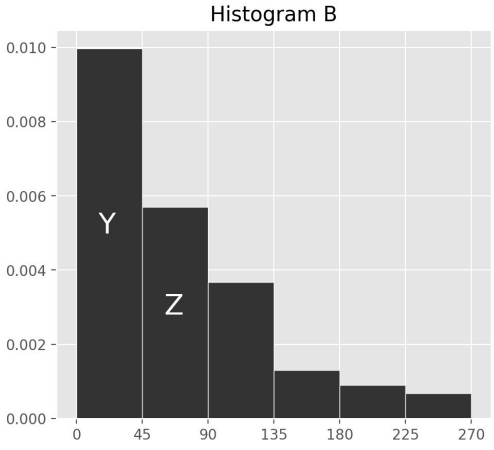
Problem 13.2

Let w, x, y, and z be the heights of bars W, X, Y, and Z, respectively. For instance, y is about 0.01.

Which of the following expressions gives the height of the bar corresponding to the $[60, 90)$ bin in Histogram A?

- $(y + z) - (w + x)$
- $(w + x) - (y + z)$
- $\frac{3}{2}(y + z) - (w + x)$
- $(y + z) - \frac{3}{2}(w + x)$
- $3(y + z) - 2(w + x)$
- $2(y + z) - 3(w + x)$
- None of the above.





Problem 14

As mentioned in the previous problem, Ashley has sample of 400 rows of `txn`. Coincidentally, in Ashley's sample of 400 transactions, the mean and standard deviation of the `"amount"` column both come out to 70 dollars.

$\frac{25}{400} = \frac{1}{16}$

Problem 14.1

Fill in the blank:

"According to Chebyshev's inequality, at most 25 transactions in Ashley's sample are above ___ dollars; the rest must be below ___ dollars."

What goes in the blank? Give your answer as an **integer**. Both blanks are filled in with the same number.

$z = 4$

$\frac{15}{16}$ of data

Chebyshev: Within z SDs of mean, get at least $1 - \frac{1}{z^2}$ prop. of data

right endpoint

mean + 4 SDs

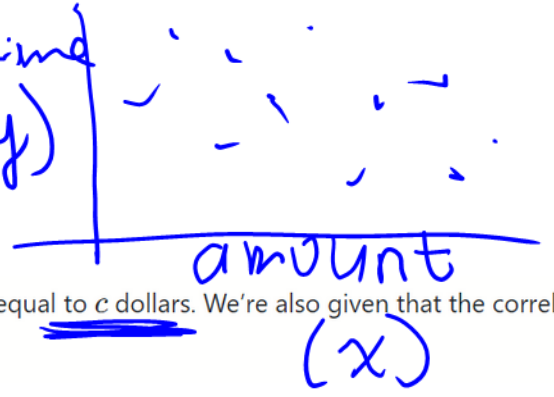
$70 + 4 \cdot 70$

$5 \cdot 70$

$\boxed{350}$

Problem 14

As mentioned in the previous problem, Ashley has sample of 400 rows of `txn`. Coincidentally, in Ashley's sample of 400 transactions, the mean and standard deviation of the `"amount"` column both come out to 70 dollars.



Now, we're given that the mean and standard deviation of the `"lifetime"` column in Ashley's sample are both equal to c dollars. We're also given that the correlation between transaction amounts and lifetime spending in Ashley's sample is $-\frac{1}{4}$.

Problem 14.3

The predicted lifetime spending, in **dollars**, of a card with a transaction amount of 280 dollars is of the form $f \cdot c$, where f is a fraction. What is the value of f? Give your answer as a simplified fraction.

$r = -1/4$

mean $x = 70$

SD $x = 70$

mean $y = c$

SD $y = c$

Predict y when $x = 280$

original units

$$m = r \cdot \frac{SD_y}{SD_x} = \frac{-\frac{1}{4} \cdot c}{70} = \frac{-1}{280} \cdot c$$

$$b = \text{mean } y - m \cdot \text{mean } x$$

$$c + \frac{1}{280} \cdot c \cdot 70 = \frac{5}{4} \cdot c$$

make prediction: $y = mx + b = \frac{-1}{280} \cdot c \cdot 280 + \frac{5}{4} c$

$$= -c + \frac{5}{4} c = \boxed{\frac{1}{4}} c$$

f

Problem 14

As mentioned in the previous problem, Ashley has sample of 400 rows of `txn`. Coincidentally, in Ashley's sample of 400 transactions, the mean and standard deviation of the `"amount"` column both come out to 70 dollars.

Problem 14.4

Suppose the intercept of the regression line, when both transaction amounts and lifetime spending are measured in **dollars**, is 40. What is the value of c? Give your answer as an integer.

original units

from previous:

$b = \text{mean } y - m \cdot \text{mean } x$

$c + \frac{1}{280} \cdot c \cdot 70 \approx \frac{5}{4} \cdot c$

$b = \frac{5}{4} \cdot c = 40$

$c = 40 \cdot \frac{4}{5}$ (8, 1)

$\boxed{c = 32}$

Problem 1

Problem 1.1

Nate's favorite number is 5. He calls a number "lucky" if it's greater than 500 or if it contains a 5 anywhere in its representation. For example, `1000.04` and `5.23` are both lucky numbers.

Complete the implementation of the function `check_lucky`, which takes in a number as a float and returns `True` if it is lucky and `False` otherwise. Then, add a column named `"is_lucky"` to `txn` that contains `True` for lucky transaction amounts and `False` for all other transaction amounts, and save the resulting DataFrame to the variable `luck`.

```
def check_lucky(x):
    return __(a)__

luck = txn.assign(is_lucky = __(b)__)
```

1. What goes in blank (a)?

2. What goes in blank (b)?

x = 5.23

str(x) = "5.23"

"5" in "5.23"

>500

(x > 500) or ("5" in str(x))

if :
    return True
else:
    return False

txn.get("amount")
.apply(check_lucky)

equivalent to

return (x > 500) or ("5" in str(x))

Problem 1

Problem 1.2

Fill in the blanks below so that `lucky_prop` evaluates to the proportion of fraudulent `"visa"` card transactions whose transaction amounts are lucky.

```
visa_fraud = __(a)__
lucky_prop = visa_fraud.__(b)__.mean()
```

1. What goes in blank (a)?
2. What goes in blank (b)?

query &

fraudulent visa lucky / # fraudulent visa

get("is_lucky")

transaction_id	is_fraud	amount	method	card	lifetime	browser	is_lucky
3061515	False	25.07	debit	visa	69306.00	chrome 63.0	T
3440724	False	19.80	debit	visa	224403.00	chrome 65.0 for android	F
3026777	False	25.00	credit	discover	75391.00	chrome 62.0	T
3122032	True	18.71	debit	visa	535891.00	chrome 63.0 for android	F
3111095	False	150.00	debit	visa	119037.00	edge 15.0	⋮
3041425	False	50.00	debit	visa	46021.00	chrome 62.0	F
3086600	False	58.42	credit	mastercard	205286.00	ie 11.0 for desktop	F
3380448	False	160.72	credit	visa	230157.00	chrome 65.0	

Problem 1

lucky-prop = # fraudulent visa lucky / # fraudulent visa

Problem 1.3

Fill in the blanks below so that `lucky_prop` is one value in the Series `many_props`.

```
many_props = luck.groupby(__(a)__).mean().get(__(b)__)
```

1. What goes in blank (a)?

2. What goes in blank (b)?

['card', 'is_fraud']

'is_lucky'

transaction_id	is_fraud	amount	method	card	lifetime	browser
3061515	False	25.07	debit	visa	69306.00	chrome 63.0
3440724	False	19.80	debit	visa	224403.00	chrome 65.0 for android
3026777	False	25.00	credit	discover	75391.00	chrome 62.0
3122032	True	18.71	debit	visa	535891.00	chrome 63.0 for android
3111095	False	150.00	debit	visa	119037.00	edge 15.0
3041425	False	50.00	debit	visa	46021.00	chrome 62.0
3086600	False	58.42	credit	mastercard	205286.00	ie 11.0 for desktop
3380448	False	160.72	credit	visa	230157.00	chrome 65.0

is_lucky F ! ! !

rows like

visa fraud

visa nonfraud

amex fraud

amex nonfraud

...

visa : prop of visa cards that are lucky

Problem 2

Consider the DataFrame `combo`, defined below.

```
combo = txn.groupby(["is_fraud", "method", "card"]).mean()
```

$2 \times 2 \times 4 = 16$

Problem 2.1

What is the maximum possible value of `combo.shape[0]`? Give your answer as an integer.

how many rows maximum?

16

Problem 10

As a senior suffering from senioritis, Weiyue has plenty of time on his hands. 1,000 times, he repeats the following process, creating 1,000 confidence intervals:

1. Collect a simple random sample of 100 rows from `txn`.
2. Resample from his sample 10,000 times, computing the mean transaction amount in each resample.
3. Create a 95% confidence interval by taking the middle 95% of resample means.

He then computes the width of each confidence interval by subtracting its left endpoint from its right endpoint; e.g. if $[2, 5]$ is a confidence interval, its width is 3. This gives him 1,000 widths. Let M be the mean of these 1,000 widths.

new samples each time

what is population here? txn DataFrame

"typical width of a confidence interval

Problem 10.1

Select the true statement below.

- About 950 of Weiyue's intervals will contain the mean transaction amount of all transactions ever.
- About 950 of Weiyue's intervals will contain the mean transaction amount of all transactions in `txn`.
- About 950 of Weiyue's intervals will contain the mean transaction amount of all transactions in the first random sample of 100 rows of `txn` Weiyue took.
- About 950 of Weiyue's intervals will contain M.