Today we'll go over solutions to last quarter's Final Exam (Fall 2024). The exam has a bookstore theme.

How familiar are you with this exam?

- A. I have not looked at it all.
- B. I have looked at it, but that's it.
- C. I have done some questions.
- D. I did the whole thing.

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Final Exam - DSC 10, Fall 2024

Full Name:	
PID:	
Seat you are in:	

Instructions:

- This exam consists of 11 questions, worth a total of 180 points.
- Write your PID in the top right corner of each page in the space provided.
- Please write **clearly** in the provided answer boxes; we will not grade work that appears elsewhere. Completely fill in bubbles and square boxes; if we cannot tell which option(s) you selected, you may lose points.

() A bubble means that you should only **select one choice**.

A square box means you should **select all that apply**.

• You may use one page of double-sided handwritten notes. Aside from this, you may not refer to any other resources or technology during the exam. No calculators!

By signing below, you are agreeing that you will behave honestly and fairly during and after this exam.

Signature:

Version A

Please do not open your exam until instructed to do so.

Important: Before proceeding, make sure to rip off the last page of this exam packet and read the data description.

Question 1 (8 pts) \rightarrow 0 0 0

a) (3 pts) Notice that bookstore has an index of "ISBN" and sales does not. Why is that?

○ There is no good reason. We could have set the index of sales to "ISBN".

 \bigcirc There can be two different books with the same "ISBN".

- "ISBN" is already being used as the index of bookstore, so it shouldn't also be used as the index of sales.
- The bookstore can sell multiple copies of the same book.

b) (2 pts) Is "ISBN" a numerical or categorical variable?

\bigcirc	numerical	\bigcirc categorica	1
\smile	mannorhour	() categorica	ч.

c) (3 pts) Which type of data visualization should be used to compare authors by median rating?

\bigcirc	scatter plot	\bigcirc line plot	\bigcirc bar chart	\bigcirc histogram
$\langle \rangle$	protection protection		() Sour chicker	

Question 2 (8 pts)

ISBN numbers for books must follow a very particular format. The first 12 digits encode a lot of information about a book, including the publisher, edition, and physical properties like page count. The 13th and final digit, however, is computed directly from the first 12 digits and is used to verify that a given ISBN number is valid.

In this question, we will define a function to compute the final digit of an ISBN number, given the first 12. First, we need to understand how the last digit is determined. To find the last digit of an ISBN number:

- 1. Multiply the first digit by 1, the second digit by 3, the third digit by 1, the fourth digit by 3, and so on, alternating 1s and 3s.
- 2. Add up these products.
- 3. Find the ones digit of this sum.
- 4. If the ones digit of the sum is zero, the final digit of the ISBN is zero. Otherwise, the final digit of the ISBN is ten minus the ones digit of the sum.

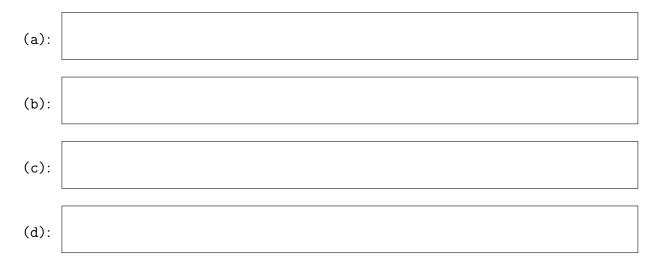
For example, suppose we have an ISBN number whose first 12 digits are 978186197271. Then the last digit would be calculated as follows:

 $9 \cdot 1 + 7 \cdot 3 + 8 \cdot 1 + 1 \cdot 3 + 8 \cdot 1 + 6 \cdot 3 + 1 \cdot 1 + 9 \cdot 3 + 7 \cdot 1 + 2 \cdot 3 + 7 \cdot 1 + 1 \cdot 3 = 118$

The ones digit is 8 so the final digit is 2.

Fill in the blanks in the function calculate_last_digit below. This function takes as input a string representing the first 12 digits of the ISBN number, and should return the final digit of the ISBN number, as an int. For example, calculate_last_digit("978186197271") should evaluate to 2.

```
def calculate_last_digit(partial_isbn_string):
    total = 0
    for i in __(a)__:
        digit = int(partial_isbn_string[i])
        if __(b)__:
            total = total + digit
        else:
            total = total + 3 * digit
        ones_digit = __(c)___
        if __(d)__:
            return 10 - ones_digit
        return 0
```



Note: this procedure may seem complicated, but it is actually how the last digit of an ISBN number is generated in real life!

Question 3 (18 pts)

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At the beginning of 2025, avid reader Michelle will join the Blind Date with a Book program sponsored by Bill's Book Bonanza. Once a month from January through December, Michelle will receive a surprise book in the mail from the bookstore. So she'll get 12 books throughout the year.

Each book is randomly chosen among those available at the bookstore (or equivalently, from among those in the **bookstore** DataFrame). Each book is equally likely to be chosen each month, and each month's book is chosen independently of all others, so repeats can occur.

Let p be the proportion of books in **bookstore** from the romance genre $(0 \le p \le 1)$.

a) (3 pts) What is the probability that the books Michelle receives in February and December are romance, while the books she receives all other months are non-romance? Your answer should be an unsimplified expression in terms of p.

b) (3 pts) What is the probability that at least one of the books Michelle receives in the last four months of the year is romance? Your answer should be an unsimplified expression in terms of p.

c) (3 pts) It is likely Michelle will receive at least one romance book and at least one non-romance book throughout the 12 months of the year. What is the probability that this does not happen? Your answer should be an unsimplified expression in terms of p.

12

d) (3 pts) Let n be the total number of rows in **bookstore**. What is the probability that Michelle does not receive any duplicate books in the first four months of the year? Your answer should be an unsimplified expression in terms of n.

givor

P(A and B and C) = P(A) × P(B givenA) PID: → P(C given AowB)

Now, consider the following lines of code which define variables i, j, and k.

```
def foo(x, y):
    if x == "rating":
        z = bookstore[bookstore.get(x) > y]
    elif x == "genre":
        z = bookstore[bookstore.get(x) == y]
    return z.shape[0]
i = foo("rating", 3)
j = foo("rating", 4)
k = foo("genre", "Romance")
```

For both questions that follow, your answer should be an unsimplified expression in terms of i, j, k only. If you do not have enough information to determine the answer, leave the answer box blank and instead fill in the bubble.

e) (3 pts) If we know in advance that Michelle's January book will have a rating greater than 3, what is the probability that the book's rating is greater than 4?

 \bigcirc Not enough information.

f) (3 pts) If we know in advance that Michelle's January book will have a rating greater than 3, what is the probability that the book's genre is romance?

 \bigcirc Not enough information.

Question 4 (22 pts) \rightarrow 0 a M

Matilda has been working at Bill's Book Bonanza since it first opened and her shifts for each week are always randomly scheduled (i.e., she does not work the same shifts each week). Due to the system restrictions at the bookstore, when Matilda logs in with her employee ID, she only has access to the history of sales made during her shifts. Suppose these transactions are stored in a DataFrame called matilda, which has the same columns as the sales DataFrame that stores all transactions.

Matilda wants to use her random sample to estimate **the mean price of books purchased with cash** at Bill's Book Bonanza. For the purposes of this question, assume Matilda only has access to matilda, and not all of sales.

a) (8 pts) Complete the code below so that cash_left and cash_right store the endpoints of an 86% bootstrapped confidence interval for the mean price of books purchased with cash at Bill's Book Bonanza.

b) (4 pts) Next, Matilda uses the data in matilda to construct a 95% CLT-based confidence interval for the same parameter.

Given that there are 400 cash transactions in **matilda** and her confidence interval comes out as [19.58, 21.58], what is the standard deviation of the prices of all cash transactions at Bill's Book Bonanza?



- c) (4 pts) Knowing the endpoints of Matilda's 95% CLT-based confidence interval can actually help us to determine the endpoints of her 86% bootstrapped confidence interval. You may also need to know the following facts:
 - stats.norm.cdf(1.1) evaluates to 0.86
 - stats.norm.cdf(1.5) evaluates to 0.93

Estimate the value of cash_left to one decimal place.

d) (3 pts) Which of the following are valid conclusions? Select all that apply.

Approximately 95% of the values in cash_means fall within the interval [19.58, 21.58].

- Approximately 95% of books purchased with cash at Bill's Book Bonanza have a price that falls within the interval [19.58, 21.58].
- The actual mean price of books purchased with cash at Bill's Book Bonanza has a 95% chance of falling within the interval [19.58, 21.58].
- None of the above.
- e) (3 pts) Matilda has been wondering whether the mean price of books purchased with cash at Bill's Book Bonanza is \$20. What can she conclude about this?
 - () The mean price of books purchased with cash at Bill's Book Bonanza is \$20.
 - \bigcirc The mean price of books purchased with cash at Bill's Book Bonanza could plausibly be \$20.
 - () The mean price of books purchased with cash at Bill's Book Bonanza is not \$20.
 - \bigcirc The mean price of books purchased with cash at Bill's Book Bonanza is most likely not \$20.

Question 5 (29 pts)

natild

Aathi is a movie enthusiast looking to dive into book-reading. He visits Bill's Book Bonanza and asks for help from the employee on duty, Matilda.

Matilda has access to the matilda DataFrame of transactions from her shifts, as well as the bookstore DataFrame of all books available in the store. She decides to advise Aathi based on the ratings of books that are popular enough to have been purchased during her shifts. Unfortunately, matilda does not contain information about the rating or genre of the books sold. To fix this, she merges the two DataFrames and keeps only the columns that she'll need.

(3 pts) Fill in the blanks in the code below so that the resulting merged DataFrame has one row for each book in matilda, and columns representing the genre and rating Samo of each such book. 65 merged = (matilda.merge(bookstore, __(a)__,

a:

BNS

(b)_ .get(["genre", "rating"])) b:

ISB Nr

b) (3 pts) In this example, matilda and merged have the same number of rows. Why is this the case? Choose the answer which is sufficient alone to guarantee the same mahlda number of rows.

> Because every book in bookstore appears exactly once in matilda. unique

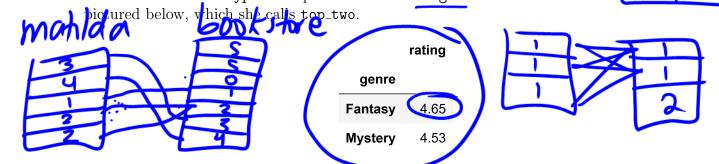
Because every book in matilda appears exactly once in bookstore.

() Because matilda has no duplicate rows

) Because bookstore has no duplicate rows.

) Because the books in matilda are a subset of the books in bookstore

Matilda uses some **babypandas** operations on the **merged** DataFrame to create the DataFrame



This DataFrame shows the two genres in merged with the highest mean rating. Note that the "rating" column in top_two represents a mean rating across many books of the same KTMP genre, not the rating of any individual book.

c) (5 pts) Write one line of code to define the DataFrame top_two as described above. It's okay if you need to write your answer on multiple lines, as long as it represents one line of code.

Based on the data in top_two, Matilda recommends that Aathi purchase a fantasy book. However, Aathi is skeptical because he recognizes that the data in merged is only a sample from the larger population of all transactions at Bill's Book Bonanza. Before he makes his purchase, he decides to do a permutation test to determine whether fantasy books have higher ratings than mystery books in this larger population.

d) (2 pts) Select the best statement of the null hypothesis for this permutation test.

○ Among all books sold at Bill's Book Bonanza, fantasy books have a higher rating than mystery books, on average.

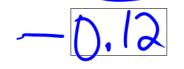
○ Among all books sold at Bill's Book Bonanza, fantasy books do not have a higher rating than mystery books, on average.

Among all books sold at Bill's Book Bonanza, fantasy books have the same rating as mystery books, on average.

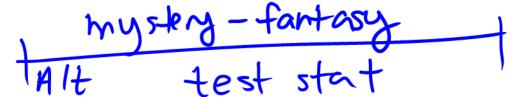
e) (2 pts) Select the best statement of the alternative hypothesis for this permutation test.

Among all books sold at Bill's Book Bonanza, fantasy books have a higher rating than mystery books, on average.

- Among all books sold at Bill's Book Bonanza, fantasy books do not have a higher rating than mystery books, on average.
- Among all books sold at Bill's Book Bonanza, fantasy books have the same rating as mystery books, on average.
- f) (3 pts) Aathi decides to use the mean rating for mystery books minus the mean rating for fantasy books as his test statistic. What is his observed value of this statistic?



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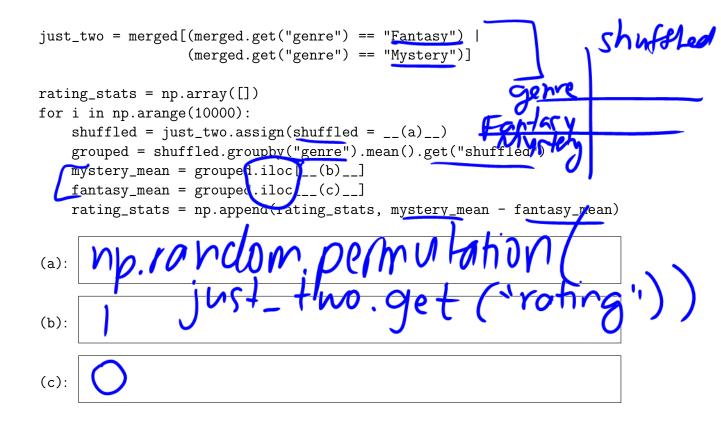
g) (3 pts) Which of the following best describes how Aathi will interpret the results of his permutation test?

 \bigcirc High values of the observed statistic will make him lean towards the alternative hypothesis.

Low values of the observed statistic will make him lean towards the alternative hypothesis.

 \bigcirc Both high and low values of the observed statistic will make him lean towards the alternative hypothesis.

h) (5 pts) Fill in the blanks in the following code to perform Aathi's permutation test.



- i) (3 pts) Aathi gets a p_value of 0.008, and he will base his purchase on this result, using the standard p-value cutoff of 0.05. What will Aathi end up doing?
 - Aathi fails to reject the null hypothesis and will purchase a fantasy book.
 - Aathi fails to reject the null hypothesis and will purchase a mystery book.

Aathi rejects the null hypothesis and will purchase a fantasy book.

• Aathi rejects the null hypothesis and will purchase a mystery book.

Question 6 (9 pts)

Bill is curious about whether his bookstore is busier on weekends (Saturday, Sunday) or weekdays (Monday, Tuesday, Wednesday, Thursday, Friday). To help figure this out, he wants to define some helpful one-line functions.

The function find_day should take in a given "date" and return the associated day of the week. For example, find_day("Saturday, December 7, 2024") should evaluate to "Saturday".

The function is_weekend should take in a given "date" and return True if that date is on a Saturday or Sunday, and False otherwise. For example, is_weekend("Saturday, December 7, 2024") should evaluate to True.

a) (4 pts) Complete the implementation of both functions below.

```
def find_day(date):
    return date.__(a)__

def weekend(date):
    return find_day(date)[__(b)__] == __(c)___
(a):
    (b):
    (c):
    (c):
```

Now, Bill runs the following line of code:

sales_day = sales.assign(weekend = sales.get("date").apply(is_weekend))

- b) (5 pts) Determine which of the following code snippets evaluates to the proportion of book purchases in sales that were made on a weekend. Select all that apply.
 - sales_day[sales_day.get("weekend")].count() / sales_day.shape[0]
 - sales[sales_day.get("weekend")].shape[0] / sales_day.shape[0]
 - sales_day.get("weekend").median()
 - sales_day.get("weekend").mean()
 -] np.count_nonzero(sales_day.get("weekend") > 0.5) / sales_day.shape[0]

Question 7 (23 pts)

Hargen is an employee at Bill's Book Bonanza who tends to work weekend shifts. He thinks that Fridays, Saturdays, and Sundays are busier than other days, and he proposes the following probability distribution of sales by day:

S	unday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
0	.2	0.1	0.1	0.1	0.1	0.2	0.2

Let's use the data in **sales** to determine whether Hargen's proposed model could be correct by doing a hypothesis test. The hypotheses are:

- **Null Hypothesis**: Sales at the bookstore are randomly drawn from Hargen's proposed distribution of days.
- Alternative Hypothesis: Sales at the bookstore are not randomly drawn from Hargen's proposed distribution of days.
- a) (4 pts) Which of the following test statistics could be used to test the given hypothesis?
 Select all that apply.

The absolute difference between the proportion of books sold on Saturday and the proposed proportion of books sold on Saturday (0.2).

The sum of the differences in proportions between the distribution of books sold by day and Hargen's proposed distribution.

The sum of the squared differences in proportions between the distribution of books sold by day and Hargen's proposed distribution.

One half of the sum of the absolute differences in proportions between the distribution of books sold by day and Hargen's proposed distribution.

We will use as our test statistic the mean of the absolute differences in proportions between the distribution of books sold by day and Hargen's proposed distribution.

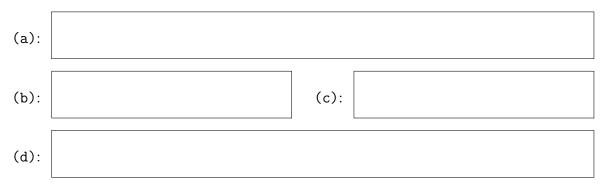
b) (3 pts) Suppose the observed distribution of books sold by day was as follows. Calculate the observed statistic in this case.

Sunday	Monday	Tuesday	Wednesday	Thursday	Friday	Saturday
0.34	0.13	0.06	0.07	0.08	0.08	0.24

observed statistic =

c) (7 pts) Let's determine the actual value of the observed statistic based on the data in sales. Assume that we have already defined a function called find_day that returns the day of the week for a given "date". For example, find_day("Saturday, December 7, 2024") evaluates to "Saturday". Fill in the blanks below so that the variable obs evaluates to the observed statistic.

```
obs = __(d)__
```

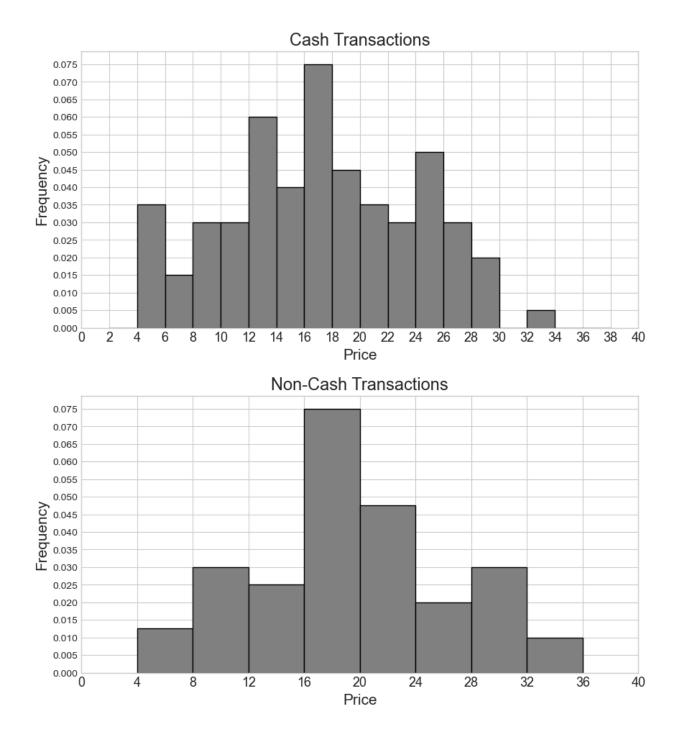


- d) (3 pts) To conduct the hypothesis test, we'll need to generate thousands of simulated day-of-the-week distributions. What will we do with all these simulated distributions?
 - Use them to determine whether Hargen's proposed distribution looks like a typical simulated distribution.
 - Use them to determine whether the observed distribution of books sold by day looks like a typical simulated distribution.
 - Use them to determine whether Hargen's proposed distribution looks like a typical observed distribution of books sold by day.
- e) (3 pts) In each iteration of the simulation, we need to collect a sample of size sales.shape[0] from which distribution?
 - Hargen's proposed distribution.
 - The distribution of data in **sales** by day of the week.
 - Our original sample's distribution.
 - \bigcirc The distribution of possible sample means.
- f) (3 pts) Suppose that obs comes out to be in the 98th percentile of simulated test statistics. Using the standard p-value cutoff of 0.05, what would Hargen conclude about his original model?

```
\bigcirc It is likely correct. \bigcirc It is plausible. \bigcirc It is likely wrong.
```

Question 8 (16 pts)

Suppose we are told that **sales** contains 1000 rows, 500 of which represent cash transactions and 500 of which represent non-cash transactions. We are given two density histograms representing the distribution of "**price**" separately for the cash and non-cash transactions.



From these two histograms, we'd like to create a single combined histogram that shows the overall distribution of "price" for all 1000 rows in sales.

a) (4 pts) How many cash transactions had a "price" of \$26 or more?



b) (4 pts) Say our combined histogram has a bin [4, 6). What will the height of this bar be in the combined histogram? Fill in the answer box or select "Not enough information."



c) (4 pts) Say our combined histogram has a bin [24, 28). What will the height of this bar be in the combined histogram? Fill in the answer box or select "Not enough information."

height =
$$\bigcirc$$
 Not enough information.

d) (4 pts) Suppose now that the histograms for cash and non-cash transactions are the same as pictured, but they now represent 600 cash transactions and 400 non-cash transactions. In this situation, say our combined histogram has a bin [12, 16). What will the height of this bar be in the combined histogram? Fill in the answer box or select "Not enough information."

$$height = \bigcirc Not enough information.$$

Question 9 (18 pts)

As in the previous problem, suppose we are told that **sales** contains 1000 rows, 500 of which represent cash transactions and 500 of which represent non-cash transactions.

This time, instead of being given histograms, we are told that the distribution of "price" for cash transactions is roughly normal, with a mean of \$14 and a standard deviation of \$2. We'll call this distribution the cash curve.

Additionally, the distribution of "price" for non-cash transactions is roughly normal, with a mean of \$22 and a standard deviation of \$4. We'll call this distribution the non-cash curve.

We want to draw a curve representing the approximate distribution of "price" for all transactions combined. We'll call this distribution the combined curve.

a) (4 pts) What is the approximate proportion of area under the cash curve between \$10 and \$12? Your answer should be a number between 0 and 1.



b) (4 pts) Fill in the blanks in the code below so that the expression evaluates to the approximate proportion of area under the **cash curve** between \$14.50 and \$17.50. Each answer should be a single number.

scipy.stats.norm.cdf(a) - scipy.stats.norm.cdf(b)



- c) (3 pts) Will the combined curve be roughly normal?
 - Yes, because of the Central Limit Theorem.
 - \bigcirc Yes, because combining two normal distributions always produces a normal distribution.
 - \bigcirc No, not necessarily.
- d) (4 pts) Fill in the blanks in the code below so that the expression evaluates to the approximate proportion of area under the **combined curve** between \$14 and \$22. Each answer should be a single number.

(scipy.stats.norm.cdf(a) - scipy.stats.norm.cdf(b)) / 2

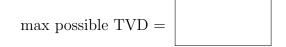


e) (3 pts) What is the approximate proportion of area under the **combined curve** between \$14 and \$22? Choose the closest answer below.

 $\bigcirc 0.47$ $\bigcirc 0.49$ $\bigcirc 0.5$ $\bigcirc 0.95$ $\bigcirc 0.97$

Question 10 (8 pts)

a) (4 pts) Suppose the bookstore DataFrame has 10 unique genres, and we are given a sample of 350 books from that DataFrame. Determine the maximum possible total variation distance (TVD) that could occur between our sample's genre distribution and the uniform distribution where each genre occurs with equal probability. Your answer should be a single number.



- **b)** (2 pts) True or False: If the sample instead had 700 books, then the maximum possible TVD would **increase**.
 - \bigcirc True \bigcirc False
- c) (2 pts) True or False: If the bookstore DataFrame had 11 genres instead of 10, the maximum possible TVD would increase.
 - \bigcirc True \bigcirc False

Question 11 (21 pts)

Dhruv works at the bookstore, and his job involves pricing new books that come in from the supplier. He prices new books based on the number of pages they have. He does this using linear regression, which he learned about in DSC 10.

To build his regression line, Dhruv gathers the following information about the distinct books currently available at the bookstore:

- The correlation between price and number of pages is 0.6.
- The mean price of all books is \$15, with a standard deviation of \$4.
- The mean number of pages of all books is 500, with a standard deviation of 200.
- a) (6 pts) Which of the following statements about Dhruv's regression line are true? Select all that apply.
 - It goes through the point (500, 15).
 - It goes through the point (200, 4).
 - \Box Its slope is equal to 0.6.
 - \Box Its *y*-intercept is equal to 9.
 - Its root mean square error is larger than the root mean square error of any other line.
 - All the books currently available at the bookstore fall on the line.
- b) (4 pts) If "The Martian" has 30 more pages than "The Simple Wild", and both books are priced according to the regression line, how much more does "The Martian" cost than "The Simple Wild"? Give your answer as a number, in dollars and cents.



c) (4 pts) A new book added to the inventory is "The Goldfinch", which has 700 pages. How much should Dhruv charge customers for this book, according to the regression line pricing model? Give your answer as a number, in dollars and cents.



- d) (3 pts) It turns out that Dhruv had an error in his regression line because he had accidentally recorded the price of one book in the data set, "Roadside Picnic", as -\$12 instead of \$12. He builds a new regression line using the correct price for "Roadside Picnic" and he finds that his new regression line has a smaller slope than before. What can we conclude about the number of pages in "Roadside Picnic" based on this information alone?
 - "Roadside Picnic" has fewer than 500 pages.
 - "Roadside Picnic" has exactly 500 pages.
 - "Roadside Picnic" has more than 500 pages.
 - \bigcirc Not enough information.
- e) (2 pts) Suppose that Dhruv originally based his regression line on a data set which has a single row for each unique book sold at Bill's Book Bonanza. If instead, he had used a dataset with one row for each copy of a book at the bookstore (and there are multiple copies of some books), would his regression line have come out the same?

 \bigcirc Yes \bigcirc No

f) (2 pts) Suppose Dhruv bootstraps his scatterplot 10,000 times and calculates a regression line for each resample. It turns out that 95% of his bootstrapped slopes fall in the interval [a, b] and 95% of his bootstrapped intercepts fall in the interval [c, d]. Does this mean that 95% of his predicted prices for a book with 500 pages fall in the interval [500a + c, 500b + d]?

 \bigcirc Yes \bigcirc No

Feel free to draw us a picture or tell us about your fondest memory from DSC 10 this quarter.

Before turning in your exam, please make sure that your PID is on every page. Congratulations on finishing DSC 10!