

DSC 190

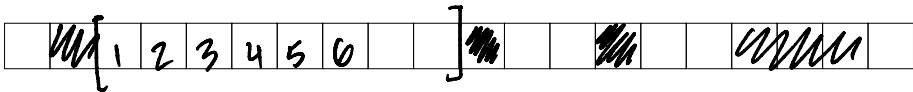
DATA STRUCTURES & ALGORITHMS

Lecture 2 | Part 1

More about Memory

Is appending an array really so slow?

- ▶ `malloc()` vs. `realloc()`



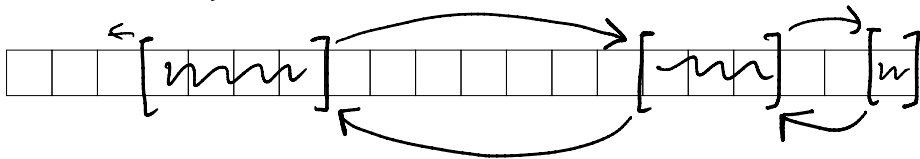
Is appending an array really so slow?

- ▶ If `realloc` doesn't copy: $\Theta(1)$.
- ▶ If `realloc` copies: $\Theta(n)$.
- ▶ Assume p is probability that `realloc` copies.
- ▶ **Expected time** is still¹ $\Theta(n)$.

¹If p doesn't depend on n .

How is empty memory found?

- Basically: a linked list.



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Lecture 2 | Part 2

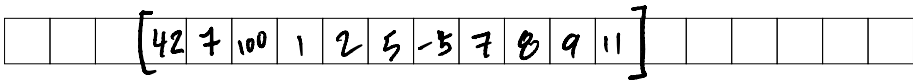
Dynamic Arrays

Motivation

- ▶ Can we have the best of both worlds?
- ▶ $\Theta(1)$ time access like an array.
- ▶ $\Theta(1)$ time append like a linked list.
- ▶ **Yes!** (sort of)

The Idea

- ▶ Allocate memory for an **underlying array**.
 - ▶ say, 512 elements
 - ▶ This is the **physical size**.
- ▶ To append element, insert into first unused slot.
 - ▶ Number of elements used is the **logical size**.
 - ▶ $\Theta(1)$ time.



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The Idea

- ▶ We'll eventually run out of unused slots.
- ▶ Fix: allocate a new underlying array whose physical size is γ times as large.
 - ▶ γ is the **growth factor**.
 - ▶ Commonly, $\gamma = 2$; i.e., double its size.
 - ▶ Takes $\Theta(k)$ time, where k is current size.

np.empty

Example



```
>>> arr = DynamicArray(initial_physical_size=4)
```

```
>>> arr.append(1)
```

```
>>> arr.append(2)
```

```
>>> arr.append(3)
```

```
>>> arr.append(4)
```

```
>>> arr.append(5)
```

} $\Theta(1)$ each

← $\Theta(n)$ time

(notebook)

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DATA STRUCTURES & ALGORITHMS

Lecture 2 | Part 3

Amortized Analysis

Analysis

- ▶ Appending takes $\Theta(1)$ time *usually*...
- ▶ ...but takes $\Theta(k)$ time when we run out of slots.
 - ▶ Where k is current size of sequence.

The Key

- ▶ Resizing is expensive, but rare.
 - ▶ If $\gamma = 2$, each new resize is twice as expensive, but happens half as often.
- ▶ Thus, the **cost per append** is small.
- ▶ **Amortize** the cost over all previous appends.

Amortized Time Complexity

- ▶ The **amortized** time for an append is:

$$T_{\text{amort}}(n) = \frac{\text{total time for } n \text{ appends}}{n}$$

- ▶ We'll see that $T_{\text{amort}}(n) = \Theta(1)$.

Amortized Analysis

total time for n appends
=
total time for **non-growing** appends
+
total time for **growing** appends

Counting Growing Appends

- ▶ Want to calculate time taken by growing appends.
- ▶ First: how many appends caused a resize?
 - ▶ β : initial physical size
 - ▶ γ : growth factor

Counting Growing Appends

- ▶ Suppose initial physical size is $\beta = 512$, and $\gamma = 2$
- ▶ Resizes occur on append #:

512, 1024, 2048, 4096, ...

- ▶ In general, resizes occur on append #:

$\beta\gamma^0, \beta\gamma^1, \beta\gamma^2, \beta\gamma^3, \dots$

Counting Growing Appends

$$2048 = 512 \cdot 2^2$$

- ▶ In a sequence of n appends, how many caused the physical size to grow?
- ▶ Simplification: Assume n is such that n th append caused a resize. Then, for some $x \in \{0, 1, 2, \dots\}$:

$$n = \beta \gamma^x$$

$$n = 512$$

$$1024$$

$$2048$$

$$1$$

$$2$$

$$3$$

- ▶ If $x = 0$ there was 1 resize; if $x = 1$ there were 2; etc.

Counting Growing Appends

- Solving for x :

$$x = \log_{\gamma} \frac{n}{\beta}$$

$$\begin{aligned}\beta &= 512 \\ \gamma &= 2 \\ n &= 700\end{aligned}$$

- Check: without assumption, $x = \lfloor \log_{\gamma} \frac{n}{\beta} \rfloor$
- Number of resizes is $\lfloor \log_{\gamma} \frac{n}{\beta} \rfloor + 1$

Counting Growing Appends

- ▶ Number of resizes is $\lfloor \log_{\gamma} \frac{n}{\beta} \rfloor + 1$
- ▶ Check with $\gamma = 2, \beta = 512, n = 400$
 - ▶ Correct # of resizes: 0
- ▶ Check with $\gamma = 2, \beta = 512, n = 1100$
 - ▶ Correct # of resizes: 2

$$\log_2 2.05 \approx 1.01$$

$$k \rightarrow \gamma^k$$

Time of Growing Appends

- ▶ How much time was taken across all appends that caused resizes?
- ▶ Assumption: resizing an array with physical size k takes time $ck = \Theta(k)$.
 - ▶ c is a constant that depends on γ .

Time of Growing Appends $\gamma = 2$ $\beta = 512$

- ▶ Time for first resize: $c\beta$. $sz = 512$
- ▶ Time for second resize: $c\gamma\beta$. 1024
- ▶ Time for third resize: $c\gamma^2\beta$. 2048
- ▶ Time for j th resize: $c\gamma^{j-1}\beta$.
- ▶ This is a **geometric progression**.

Time of Growing Appends

- ▶ Time for j th resize: $c\gamma^{j-1}\beta$.
- ▶ Suppose there are r resizes.
- ▶ Total time:

$$c\beta \sum_{j=1}^r \gamma^{j-1} = c\beta \sum_{j=0}^{r-1} \gamma^j$$

Recall: Geometric Sum

- From some class you've taken:

$$\sum_{p=0}^N x^p = \frac{1 - x^{N+1}}{1 - x}$$

- Example:

$$1 + 2 + 4 + 8 + 16 = \sum_{p=0}^4 2^p = \frac{1 - 2^5}{1 - 2} = 31$$

x=2

Time of Growing Appends

► Total time:

$$c\beta \sum_{j=0}^{r-1} \gamma^j = c\beta \frac{1 - \gamma^r}{1 - \gamma}$$

$$\sum_{p=0}^N x^p = \frac{1 - x^{N+1}}{1 - x}$$

$$N = r - 1$$

$$p = j$$

$$\gamma = x$$

Time of Growing Appends

- ▶ Remember: in n appends there are $r = \lfloor \log_{\gamma} \frac{n}{\beta} \rfloor + 1$ resizes.
- ▶ Total time:

$$\begin{aligned} c\beta \frac{1 - \gamma^r}{1 - \gamma} &= c\beta \frac{1 - \gamma^{\lfloor \log_{\gamma} \frac{n}{\beta} \rfloor + 1}}{1 - \gamma} \\ &= \Theta(n) \end{aligned}$$

Amortized Analysis

total time for n appends

=

total time for **non-growing** appends

+

$\Theta(n)$ \leftarrow total time for **growing** appends

Time of Non-Growing Appends

- ▶ In a sequence of n appends, how many are **non-growing**?

$$n - \left(\lfloor \log_{\gamma} \frac{n}{\beta} \rfloor + 1 \right) = \Theta(n)$$

- ▶ Time for one such append: $\Theta(1)$.
- ▶ Total time: $\Theta(n) \times \Theta(1) = \Theta(n)$.

Amortized Analysis

total time for n appends

=

$\Theta(n)$ \leftarrow total time for **non-growing** appends

+

$\Theta(n)$ \leftarrow total time for **growing** appends

$$\Theta(n)$$

Amortized Time Complexity

- The **amortized** time for an append is:

$$\begin{aligned} T_{\text{amort}}(n) &= \frac{\text{total time for } n \text{ appends}}{n} \\ &= \frac{\Theta(n)}{n} \\ &= \Theta(1) \end{aligned}$$

Dynamic Array Time Complexities

- ▶ Retrieve k th element: $\Theta(1)$

1 2 3

- ▶ Append/pop element

- ▶ $\Theta(1)$ best case
- ▶ $\Theta(n)$ worst case (where n = current size)
- ▶ $\Theta(1)$ amortized

- ▶ Insert/remove in middle: $\Theta(k)$ $k = n - i$

- ▶ May or may not need resize, still $\Theta(n)$!

1 2 3 5 6

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DATA STRUCTURES & ALGORITHMS

Lecture 2 | Part 4

Practicalities

Advantages

- ▶ Great cache performance (it's an array).
- ▶ Fast access.
- ▶ Don't need to know size in advance of allocation.

Downsides

- ▶ Wasted memory.
- ▶ Expensive deletion in middle.

Implementations

- ▶ Python: `list`
- ▶ C++: `std::vector`
- ▶ Java: `ArrayList`

Exercise

Why do we need `np.array`? Python's `list` is a dynamic array, isn't that better?

In defense of `np.array`

- ▶ Memory savings are one reason. *1, 2, 3, 4, 5*
- ▶ Bigger reason: using Python's `list` to store numbers does not have good **cache** performance.

