# DSC 190

## DATA STRUCTURES & ALGORITHMS

## Today's Lecture

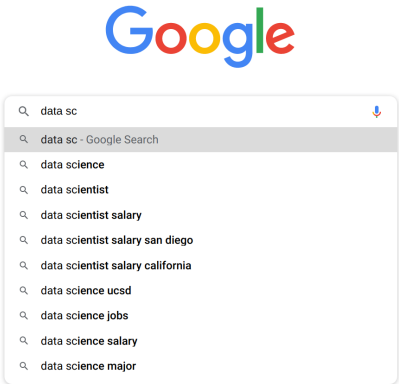
# String Data Structures

- One of the themes of this quarter:
- If you're doing something once, use an algorithm.
- If you're doing it over and over, use an appropriate data structure.

# String Data Structures

- Over the next two lectures, we'll look at data structures for strings.
- Today: **tries** for efficient repeated **prefix queries**.

# Autocompletion



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## DATA STRUCTURES & ALGORITHMS

## Tries

# Trie

- A data structure for storing strings.
- Pronounced “try”, short for “re**trie**val”.
- Supports fast **prefix query** and **membership query**.

# Prefixes

- A **prefix** `p` of a string `s` is a contiguous slice of the form `s[0:t]`, for some *t*.

- Examples:
  - `"test"` is a prefix of `"testing"`
  - `"te"` is a prefix of `"testing"`
  - `"sa"` is a prefix of `"san diego"`
  - `"di"` is **not** a prefix of `"san diego"`

# Prefix Query

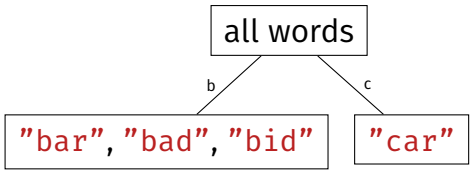
- **Given**: a collection of *n* strings and a prefix, `p`.
- **Find**: all strings in the collection for which `p` is a prefix.
- Example:
  - `"bar", "bad", "bid", "car"`
  - `p = "ba"`

# Brute Force

- Loop over each of $n$ strings, compare against prefix p.
- Worst-case time: $\Theta(n \cdot |p|)$

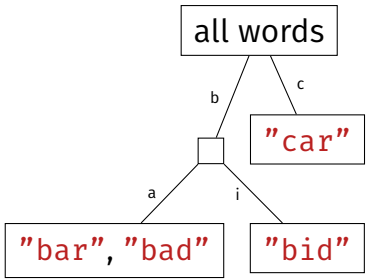
# Trie: Motivation

"bar", "bad", "bid", "car"



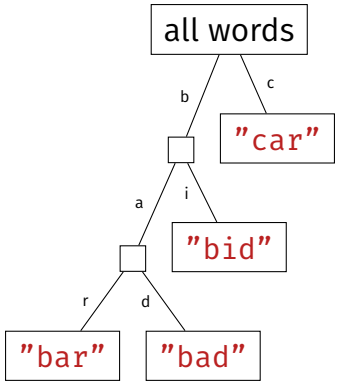
# Trie: Motivation

"bar", "bad", "bid", "car"



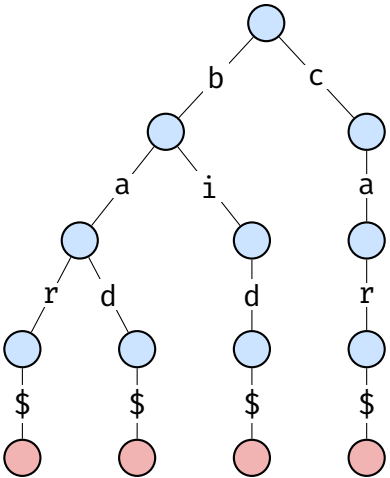
# Trie: Motivation

"bar", "bad", "bid", "car"



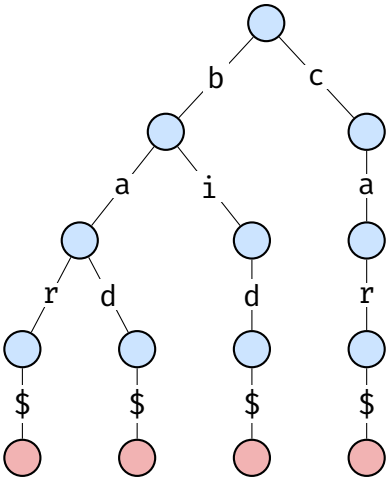
bar, bad, bid, car

# Tries




- Internal nodes represent prefixes.
- Leaf nodes represent full words.
- Edges are characters.
- Words are encoded as paths.

# Sentinels




- $ is a **sentinel**.
- It is different from the dollar sign character.
- It marks the end of a word.
- Used to show that `"bar"` in trie, but `"ba"` not.

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## DATA STRUCTURES & ALGORITHMS

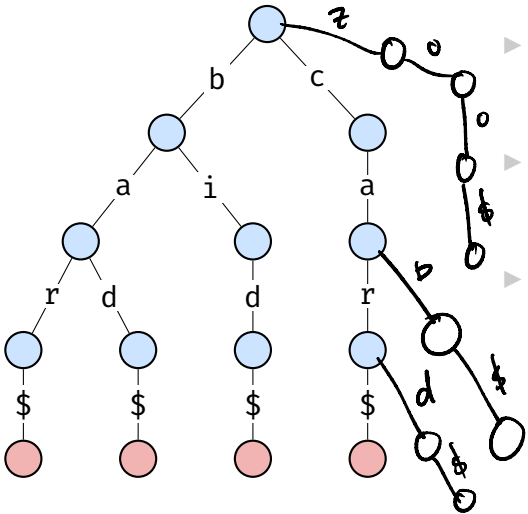
## Implementing Tries

# Representation

- Each node has a hash table / array mapping characters to a child nodes.
- Sentinel represented with a singleton object?

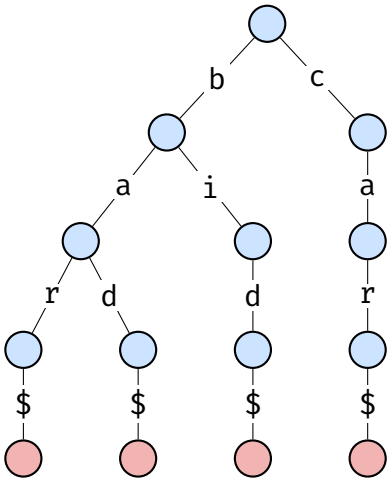
```
END_OF_STRING = object()

class TrieNode:

    def __init__(self):
        self.children = {}
```

# Insertion




- “Walk” down tree, creating edges and nodes as necessary.
- When no more letters left, add sentinel.
- Example: insert `"cab"`, `"card"`, `"zoo"`

# Insertion (Recursive)




.insert("b")

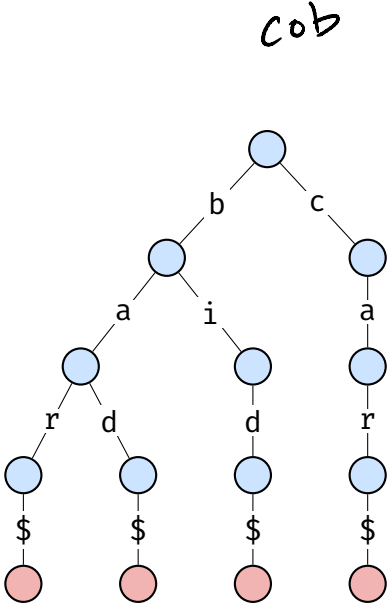
- Suppose we `.insert(s)` on root node.
- If `s[0]` not in `self.children`, create a new node.
- Otherwise, let child be `self.children[s[0]]`.
- Recursively insert `s[1:]` into child.

s[1:]

```
def insert(self, s, start=0, stop=None):
    """Insert s[start:stop] into the trie."""
    if stop is None:
        stop = len(s)

    if start >= stop:
        self.children[END_OF_STRING] = TrieNode()
        return

    if s[start] not in self.children:
        self.children[s[start]] = TrieNode()

    child = self.children[s[start]]
    child.insert(s, start + 1, stop)
```

# Insertion Time Complexity

- $\Theta(|w|)$ time, where $w$ is the string inserted.
- No matter how many elements in trie!




# Walk

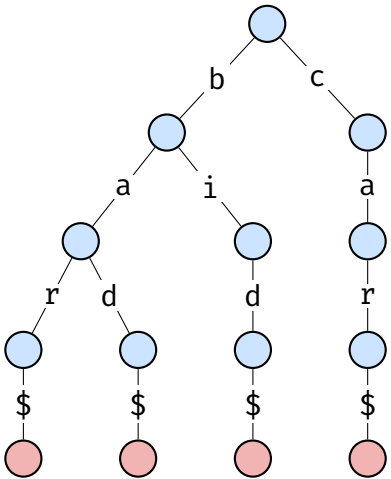
- Useful operation.
- Given a prefix, “walk” down tree.
- If we “fall off”, raise error.
- Otherwise, return last node seen.
- Examples: `"ba"`, `"bo"`

```
def walk(self, s, start=0, stop=None):
    """Walk the trie following s[start:stop].
    Raises ValueError if falls off tree.
    Returns last node encountered otherwise."""
    if stop is None:
        stop = len(s)

    if start >= stop:
        return self

    if s[start] not in self.children:
        raise ValueError('Fell off tree.')
    else:
        child = self.children[s[start]]
        return child.walk(s, start + 1, stop)
```

# Walk Time Complexity

- Worst-case $\Theta(|p|)$ time, where $p$ is the prefix searched.
- No matter how many elements in trie!

# Membership Query



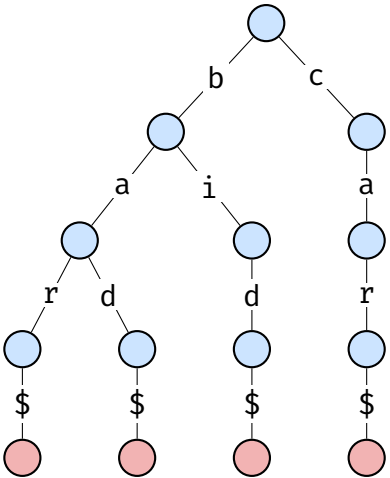

- Given `p`, return True/False if `p` in collection.
- “Walk” down tree.
- If we “fall off”, return False.
- If not, check that sentinel in children.
- Examples: `"ba"`, `"bad"`

```
def membership_query(self, s, start=0, stop=None):
    """Determine if s[start:stop] is in trie."""
    try:
        node = self.walk(s, start, stop)
    except ValueError:
        return False

    return END_OF_STRING in node.children
```

# Membership Query Time Complexity

- Worst-case $\Theta(|w|)$ time, where $w$ is the prefix searched.
- No matter how many elements in trie!

# Produce




- Goal: generate all words in subtrie.
- Perform a DFS, keeping track of letters along path.
- If we find a sentinel, print path.

```
def produce(self, pathchars=''):
    """Generate the words in the trie."""
    for letter, child in self.children.items():
        if letter is END_OF_STRING:
            yield pathchars
        else:
            yield from child.produce(pathchars + letter)
```

bad bid

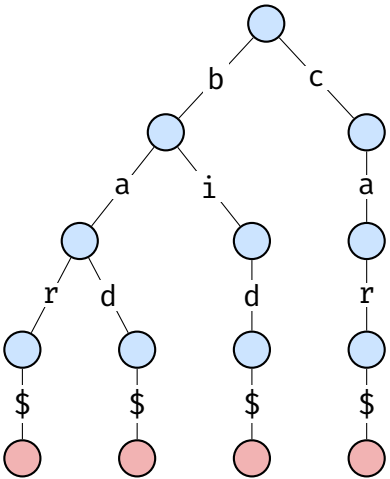
# Produce Time Complexity

O(cn)

- Worst-case $\Theta(\ell)$ time, where $\ell$ is total length of all strings stored in the `trie`.
- If length strings is considered a constant, this is $\Theta(n)$.

# of strings

# Prefix Query (Complete)




- Given `p`, return all completions.
- “Walk” down tree.
- If we “fall off”, return empty list..
- If not, produce all nodes in subtrie.
- Examples: `"ba"`, `"bad"`

```
def complete(self, prefix):
    try:
        node = self.walk(prefix)
    except ValueError:
        return []
    return list(node.produce())
```

# Prefix Query Time Complexity

- Worst-case $\Theta(|p| + \ell_p)$ time, where $p$ is the prefix searched and $\ell_p$ is the total length of all matches.
- If length is considered constant, this is $\Theta(|p| + z)$, where $z$ is number of matches.

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## Demo