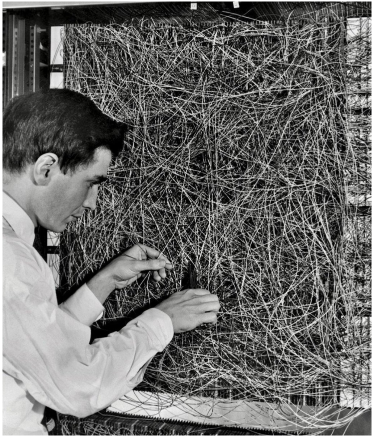
DSC 190

Machine Learning: Representations

Lecture 3 | Part 1

An Embarrassment for the Perceptron

The Perceptron



The Perceptron

- The perceptron uses a **linear prediction function**:

$$
\begin{aligned}
H(\vec{x}) &= w_0 + w_1 x_1 + w_2 x_2 + \ldots + w_d x_d \\
&= w_0 + \sum_{i=1}^{d} w_i x_i \\
&= \vec{w} \cdot \text{Aug}(\vec{x})
\end{aligned}
$$

- Trained using the **perceptron loss**.

Linear Decision Functions

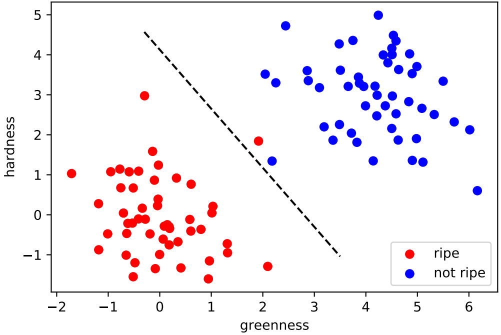
- A linear prediction function H outputs a number.
- What if classes are +1 and -1?
- Can be turned into a **decision function** by taking:

$$\operatorname{sign}(H(\vec{x}))$$

- **Decision boundary** is where $H = 0$
  - Where the sign switches from positive to negative.

Decision Boundaries

- A linear decision function's decision boundary is linear.
  - A line, plane, hyperplane, etc.



1958 New York Times...

NEW NAVY DEVICE LEARNS BY DOING

Psychologist Shows Embryo of Computer Designed to Read and Grow Wiser

WASHINGTON, July 7 (UPI) —The Navy revealed the embryo of an electronic computer today that it expects will be able to walk, talk, see, write, reproduce itself and be conscious of its existence.

The embryo—the Weather Bureau's $2,000,000 "704" computer—learned to differentiate between right and left after fifty attempts in the Navy's demonstration for newsmen.

The service said it would use this principle to build the first of its Perceptron thinking machines that will be able to read and write. It is expected to be finished in about a year at a cost of $100,000.

Dr. Frank Rosenblatt, designer of the Perceptron, conducted the demonstration. He said the machine would be the first device to think as the human brain. As do human beings, Perceptron will make mistakes at first, but will grow wiser as it gains experience, he said.

Dr. Rosenblatt, a research psychologist at the Cornell Aeronautical Laboratory, Buffalo, said Perceptrons might be fired to the planets as mechanical space explorers.

Without Human Controls

The Navy said the perceptron would be the first non-living mechanism "capable of receiving, recognizing and identifying its surroundings without any human training or control."

The "brain" is designed to remember images and information it has perceived itself. Ordinary computers remember only what is fed into them on punch cards or magnetic tape.

Later Perceptrons will be able to recognize people and call out their names and instantly translate speech in one language to speech or writing in another language, it was predicted.

Mr. Rosenblatt said in principle it would be possible to build brains that could reproduce themselves on an assembly line and which would be conscious of their existence.

In today's demonstration, the "704" was fed two cards, one with squares marked on the left side and the other with squares on the right side.

Learns by Doing

In the first fifty trials, the machine made no distinction between them. It then started registering a "Q" for the left squares and "O" for the right squares.

Dr. Rosenblatt said he could explain why the machine learned only in highly technical terms. But he said the computer had undergone a "self-induced change in the wiring diagram."

The first Perceptron will have about 1,000 electronic "association cells" receiving electrical impulses from an eye-like scanning device with 400 photo-cells. The human brain has 10,000,000,000 responsive cells, including 100,000,000 connections with the eyes.

An Example: Parking Predictor

- **Task**: Predict (yes / no): Is there parking available at UCSD right now?
- What training data to collect? What features?

Useful Features

- Time of day?
- Day’s high temperature?
- ...

Exercise

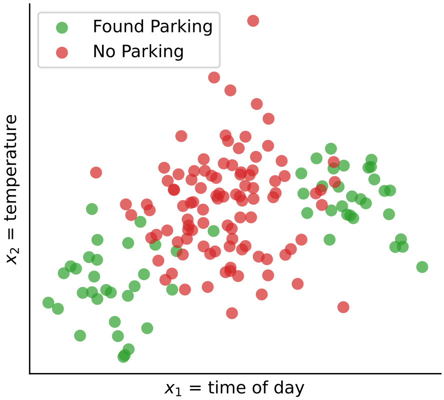
Imagine a scatter plot of the training data with the two features:

- x_1 = time of day
- x_2 = temperature

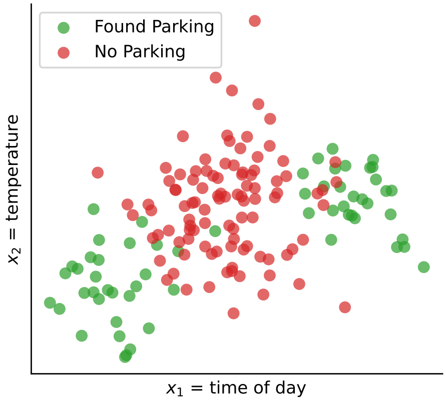
"yes" examples are green, "no" are red.

What does it look like?

Parking Data



Uh oh



- A linear decision function won’t work.
- A perceptron (or linear SVM, logistic regression model, etc.) won’t capture the trend
- What do we do?

Today's Question

- How do we learn non-linear patterns using linear prediction functions?

DSC 190

Machine Learning: Representations

Lecture 3 | Part 2

Basis Functions

Representations

- We **represented** the data with two features: time and temperature

- In this **representation**, the trend is **nonlinear**.
    - There is no good linear decision function
    - Learning is “difficult”.

Idea

- **Idea**: We'll make a new **representation** by creating **new features** from the **old features**.
- The "right" representation makes the problem easy again.
- What new features should we create?

New Feature Representation

- Linear prediction functions[1] work well when relationship is linear
  - When *x* is small we should predict -1
  - When *x* is large we should predict +1

- But parking's relationship with time is not linear:
  - When time is small we should predict +1
  - When time is medium we should predict -1
  - When time is large we should predict +1

[1] Remember: they are weighted votes.

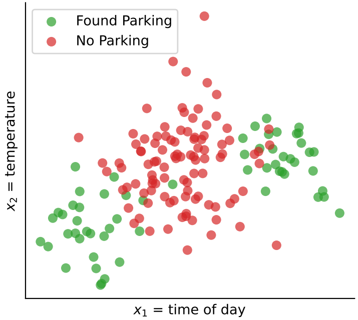
Exercise

How can we “transform” the time of day x_1 to create a new feature x_1' satisfying:

- When x_1' is small, we should predict -1
- When x_1' is large, we should predict +1

What about the temperature, x_2?

Idea



- Transform “time” to “absolute time until/since Noon”
- Transform “temp.” to “absolute difference between temp. and 72°”

Basis Functions

- We will transform:
  - the time, x_1, to $|x_1 - \text{Noon}|$
  - the temperature, x_2, to $|x_2 - 72^\circ|$
- Formally, we've designed non-linear **basis functions**:

$$\varphi_1(x_1, x_2) = |x_1 - \text{Noon}|$$
$$\varphi_2(x_1, x_2) = |x_2 - 72^\circ|$$

- In general a basis function φ maps $\mathbb{R}^d \to \mathbb{R}$

Feature Mapping

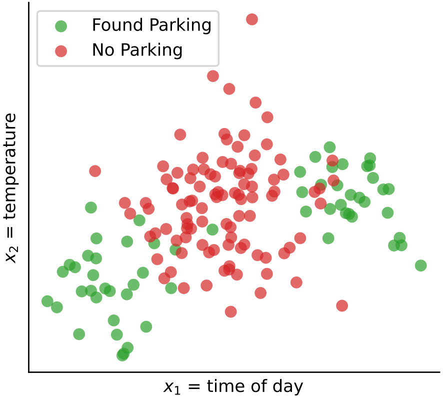
- Define $\vec{\varphi}(\vec{x}) = (\varphi_1(\vec{x}), \varphi_2(\vec{x}))^T$. $\vec{\varphi}$ is a **feature map**
  - Input: vector in “old” representation
  - Output: vector in “new” representation

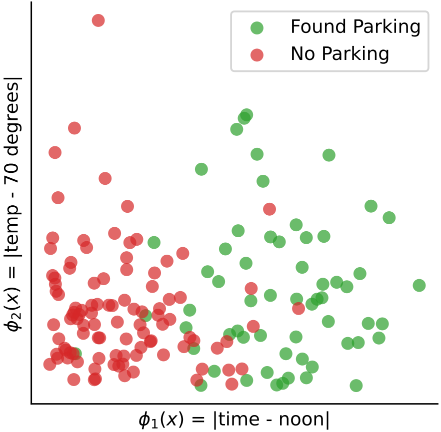
- Example:

$$\vec{\varphi}((10\text{a.m.}, 75^\circ)^T) = (2 \text{ hours}, 3^\circ)^T$$

- $\vec{\varphi}$ maps raw data to a **feature space**.

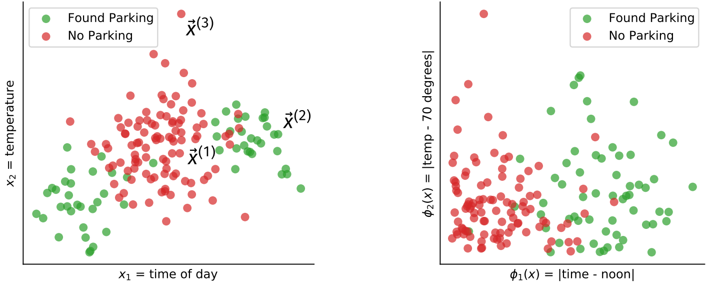
Feature Space, Visualized



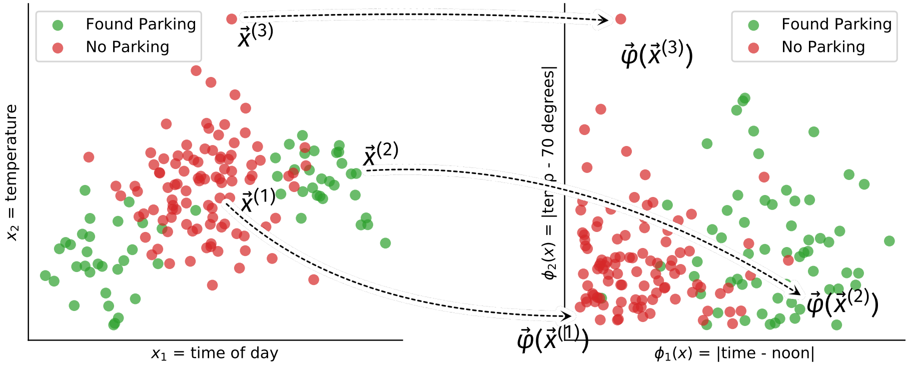


Exercise

Where does $\vec{\varphi}$ map $\vec{x}^{(1)}$, $\vec{x}^{(2)}$, and $\vec{x}^{(3)}$?



Solution



Found Parking
No Parking
$\vec{x}^{(3)}$
$\vec{x}^{(2)}$
$\vec{x}^{(1)}$
x_2 = temperature
x_1 = time of day
$\vec{\varphi}(\vec{x}^{(3)})$
$\vec{\varphi}(\vec{x}^{(2)})$
$\vec{\varphi}(\vec{x}^{(1)})$
Found Parking
No Parking
$\phi_2(x)$ = |temp - 70 degrees|
$\phi_1(x)$ = |time - noon|

After the Mapping

- The basis functions φ_1, φ_2 give us our "new" features.
- This gives us a new **representation**.
- In this representation, learning (classification) is easier.

Training

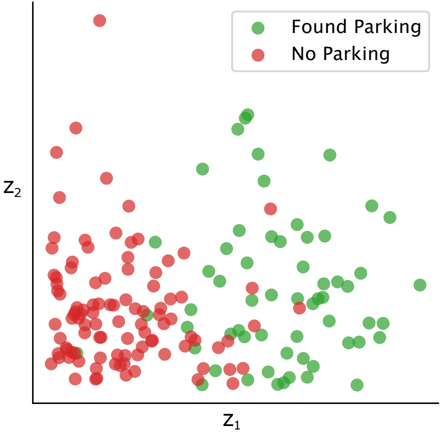
- Map each training example $\vec{x}^{(i)}$ to feature space, creating new training data:

$$\vec{z}^{(1)} = \vec{\varphi}(\vec{x}^{(1)}), \quad \vec{z}^{(2)} = \vec{\varphi}(\vec{x}^{(2)}), \quad \ldots, \quad \vec{z}^{(n)} = \vec{\varphi}(\vec{x}^{(n)})$$

- Fit linear prediction function H in usual way:

$$H_f(\vec{z}) = w_0 + w_1 z_1 + w_2 z_2 + \ldots + w_d z_d$$

Training Data in Feature Space



Prediction

- If we have $\vec{z}$ in feature space, prediction is:

$$H_f(\vec{z}) = w_0 + w_1 z_1 + w_2 z_2 + \ldots + w_d z_d$$

Prediction

- But if we have $\vec{x}$ from original space, we must "convert" $\vec{x}$ to feature space first:

$$
\begin{aligned}
H(\vec{x}) &= H_f(\vec{\varphi}(\vec{x})) \\
&= H_f(\,(\varphi_1(\vec{x}),\ \varphi_2(\vec{x}),\ \ldots,\ \varphi_d(\vec{x}))^T\,) \\
&= w_0 + w_1\varphi_1(\vec{x}) + w_2\varphi_2(\vec{x}) + \ldots + w_d\varphi_d(\vec{x})
\end{aligned}
$$

Overview: Feature Mapping

- A basis function can involve any/all of the original features:

$$\varphi_3(\vec{x}) = x_1 \cdot x_2$$

- We can make more basis functions than original features:

$$\vec{\varphi}(\vec{x}) = (\, \varphi_1(\vec{x}),\ \varphi_2(\vec{x}),\ \varphi_3(\vec{x})\,)^T$$

Overview: Feature Mapping

1. Start with data in original space, $\mathbb{R}^d$.

2. Choose some basis functions, $\varphi_1, \varphi_2, \ldots, \varphi_{d'}$

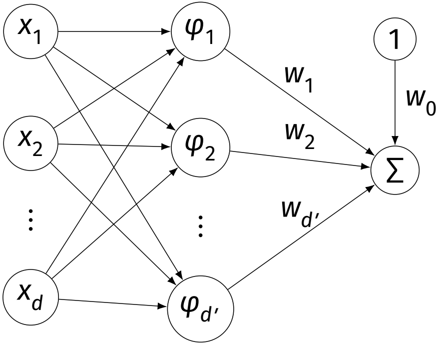
3. Map each data point to **feature space** $\mathbb{R}^{d'}$:

$$\vec{x} \mapsto (\varphi_1(\vec{x}), \varphi_2(\vec{x}), \ldots, \varphi_{d'}(\vec{x}))^t$$

4. Fit linear prediction function in new space:

$$H(\vec{x}) = w_0 + w_1 \varphi_1(\vec{x}) + w_2 \varphi_2(\vec{x})$$

$$H(\vec{x}) = w_0 + w_1\varphi_1(\vec{x}) + w_2\varphi_2(\vec{x})$$



Today's Question

- Q: How do we learn non-linear patterns using linear prediction functions?
- A: Use non-linear basis functions to map to a feature space.

DSC 190

Machine Learning: Representations

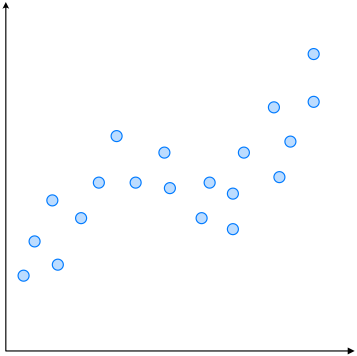
Lecture 3 | Part 3

Basis Functions and Regression

By the way...

- You've (probably) seen basis functions used before.
- Linear regression for non-linear patterns in DSC 40A.

Example



Fitting Non-Linear Patterns

- Fit function of the form

$$H(x) = w_0 + w_1 x + w_2 x^2 + w_3 x^3 + w_4 x^4$$

- Linear function of $\vec{w}$, non-linear function of x.

The Trick

- Treat x, x^2, x^3, x^4 as **new** features.
- Create design matrix:

$$X = \begin{pmatrix} 1 & x_1 & x_1^2 & x_1^3 & x_1^4 \\ 1 & x_2 & x_2^2 & x_2^3 & x_2^4 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & x_n & x_n^2 & x_n^3 & x_n^4 \end{pmatrix}$$

- Solve $X^T X \vec{w} = X^T \vec{w}$ for $\vec{w}$, as usual.
- Works for more than just polynomials.

Another View

- We have changed the representation of a point:

$$x \mapsto (x, x^2, x^3, x^4)$$

- Basis functions:

$$\varphi_1(x) = x \quad \varphi_2(x) = x^2 \quad \varphi_3(x) = x^3 \quad \varphi_4(x) = x^4$$

DSC 190

Machine Learning: Representations

Lecture 3 | Part 4

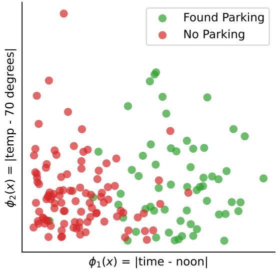
A Tale of Two Spaces

A Tale of Two Spaces

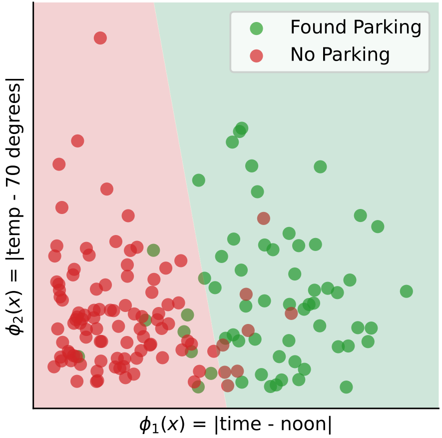
- The **original space**: where the raw data lies.
- The **feature space**: where the data lies after feature mapping $\vec{\varphi}$
- Remember: we fit a linear prediction function in the **feature space**.

Exercise

- In **feature space**, what does the decision boundary look like?
- What does the prediction function surface look like?

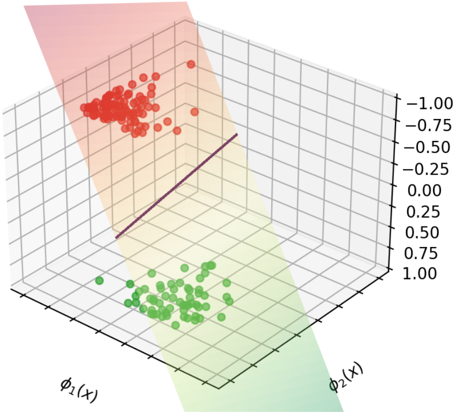


Decision Boundary in Feature Space[2]



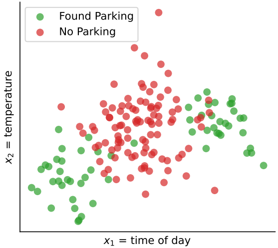
[2] Fit by minimizing square loss

Prediction Surface in Feature Space

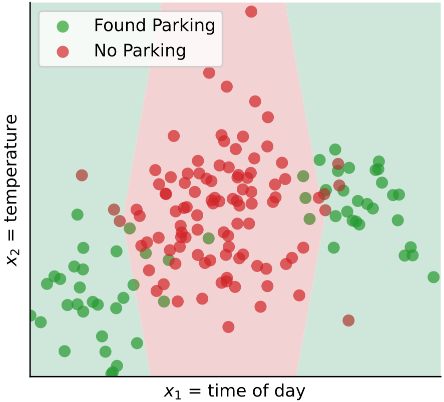


Exercise

- In the **original space**, what does the decision boundary look like?
- What does the prediction function surface look like?

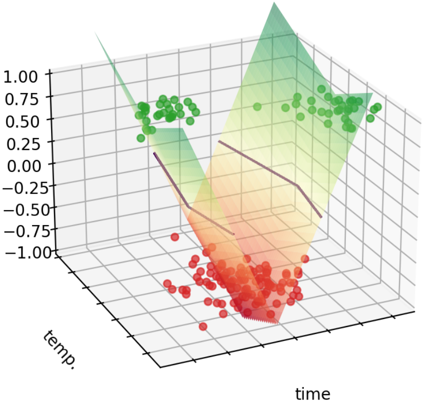


Decision Boundary in Original Space[3]



[3] Fit by minimizing square loss

Prediction Surface in Original Space



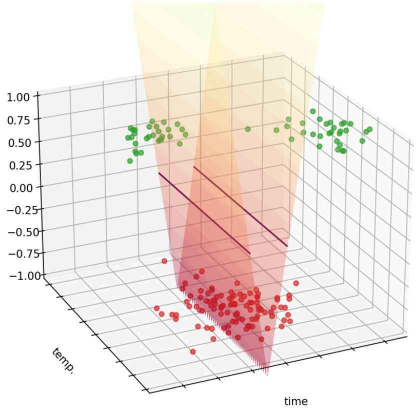
Insight

- H is a sum of basis functions, φ_1 and φ_2.
    - $H(\vec{x}) = w_0 + w_1\varphi_1(\vec{x}) + w_2\varphi_2(\vec{x})$

- The prediction surface is a sum of other surfaces.

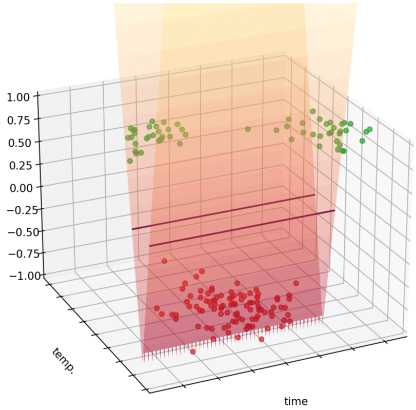
- Each basis function is a “building block”.

Visualizing the Basis Function φ_1



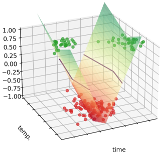
- $w_0 + w_1 |x_1 - \text{noon}|$

Visualizing the Basis Function φ_2



- $w_0 + w_2|x_2 - 72^{\circ}|$

Visualizing the Prediction Surface



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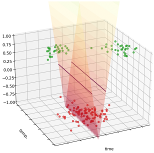

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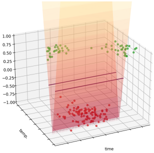
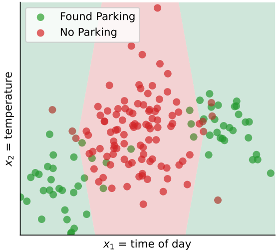


Exercise

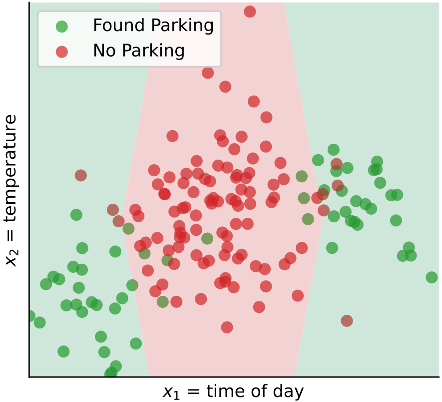
The decision boundary has a single "pocket" where it is negative. Can it have more than one, assuming we use basis functions of the same form? What if we use more than two basis functions?



Answer: No!

- Recall: the sum of **convex** functions is **convex**.
- Each of our basis functions is convex.
- So the prediction surface will be convex, too.
- Limited in what patterns they can classify.

View: Function Approximation



- Find a function that is ≈ 1 near green points and ≈ −1 near red points.

What’s Wrong?

- We’ve discovered how to learn non-linear patterns using linear prediction functions.
  - Use non-linear basis functions to map to a feature space.
- Something should bug you, though…