Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. You must work in a group of 2 to 4 students for at least 50 minutes to get credit for this assignment. It's best to join a discussion section if possible.

**One person** from each group should submit your solutions to Gradescope by 11:59pm on Monday. Make sure to **tag all group members** so everyone gets credit. This worksheet won't be graded on correctness, but rather on good-faith effort. Even if you don't solve any of the problems, you should include some explanation of what you thought about and discussed, so that you can get credit for spending time on the assignment.

# 1 Least Squares Regression

Recall that the least squares solutions to the problem of fitting a straight line, $H(x) = w_0 + w_1 x$, to the data $(x_i, y_i)$ are:

$$w_1^* = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^{n}(x_i - \bar{x})^2}$$

$$w_0^* = \bar{y} - w_1^* \bar{x}$$

where $\bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$ and $\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i$.

**Problem 1.**

Show that if all the $y$-values in a data set are multiplied by the same constant $c$, then the slope of the least squares regression line is also multiplied by $c$.

Now, if instead of multiplying by constant $c$, we add all $y$-values by the same constant $c$, what according to you will be the change in the slope. Provide a proof for your answer.

**Problem 2.**

Consider the data set given below. Fit a line $y = a + bx$ by the method of least squares, where $x$ is the predictor variable and $y$ is the response variable, and sketch it on the plot. Fit a line $x = c + dy$ by the method of least squares, where $y$ is the predictor variable and $x$ is the response variable, and sketch it on the plot. (You can use a calculator or Python to find $a$, $b$, $c$, and $d$.)

Are the lines the same or different? Explain why.

| x | 0.35 | 1.56 | -0.55 | 0.70 | 1.86 | -0.68 | -0.30 | -1.18 | 0.60 | -1.75 |
|---|------|------|-------|------|------|-------|-------|-------|------|-------|
| y | 0.30 | 1.56 | -0.45 | 0.55 | 1.58 | -0.88 | -0.72 | -0.81 | 0.65 | -1.59 |

**Data Points**
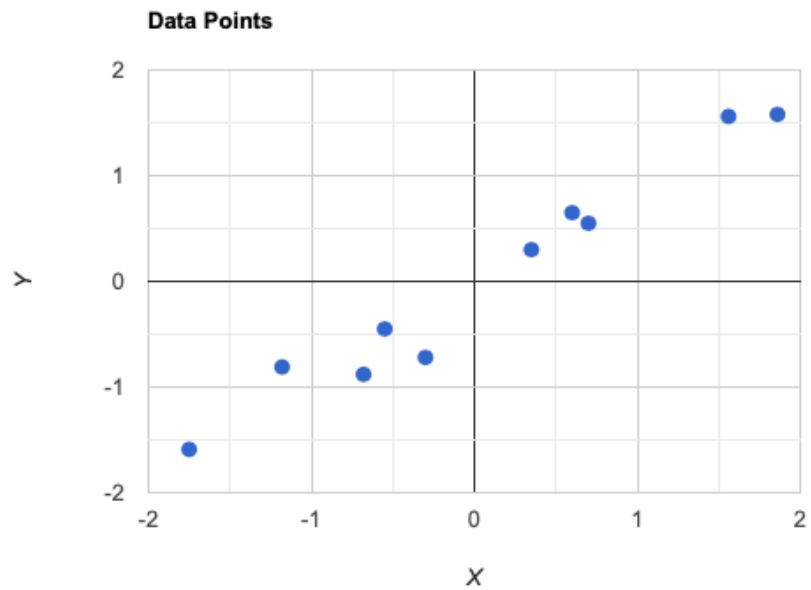


**Problem 3.**

Consider the problem of fitting a function of the form $H(x) = b_0 + b_1 \cos(x)$ to the data $(x_1, y_1), \ldots, (x_n, y_n)$. What are the least squares solutions for $b_0$ and $b_1$?

**Hint:** While this looks different than what we've studied in lecture, it turns out that it's quite similar. What if we define a change of variables, such that $z_i = \cos(x_i)$?