Lecture 18 – Foundations of Probability

DSC 40A, Fall 2022 @ UC San Diego
Mahdi Soleymani, with help from **many others**

Agenda

- ▶ Conditional probability.

Conditional probability

Conditional probability

- ▶ The probability of an event may **change** if we have additional information about outcomes.

- ▶ Starting with the multiplication rule, $P(A \cap B) = P(A)P(B|A)$, we have that

$$P(B|A) = \frac{P(A \cap B)}{P(A)}$$

  assuming that $P(A) > 0$.

Example: families

Suppose a family has two pets. Assume that it is equally likely that each pet is a dog or a cat. Consider the following two probabilities:

1. The probability that both pets are dogs given that **the oldest is a dog**.

2. The probability that both pets are dogs given that **at least one of them is a dog**.

Example: families

Let's compute the probability that both pets are dogs given
that **the oldest is a dog**.

Example: families

Let's now compute the probability that both pets are dogs given that **at least one of them is a dog**.

Example: dominoes (source: 538)

In a set of dominoes, each tile has two sides with a number of dots on each side: zero, one, two, three, four, five, or six. There are 28 total tiles, with each number of dots appearing alongside each other number (including itself) on a single tile.
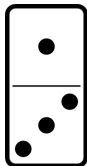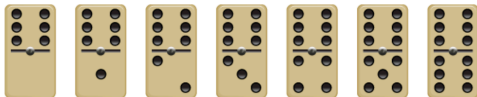
Example: dominoes (source: 538)

Question 1: What is the probability of drawing a "double" from a set of dominoes — that is, a tile with the same number on both sides?

Example: dominoes (source: 538)

Question 2: Now your friend picks a random tile from the set and tells you that at least one of the sides is a 6. What is the probability that your friend's tile is a double, with 6 on both sides?

Example: dominoes (source: 538)

Question 3: Now you pick a random tile from the set and uncover only one side, revealing that it has six dots. What is the probability that this tile is a double, with six on both sides?
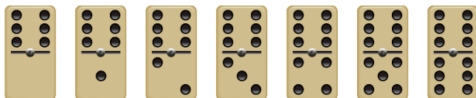
Simpson's Paradox (source: nih.gov)

	Treatment A	Treatment B
Small kidney stones	81 successes / 87 (93%)	234 successes / 270 (87%)
Large kidney stones	192 successes / 263 (73%)	55 successes / 80 (69%)
Combined	273 successes / 350 (78%)	289 successes / 350 (83%)

Discussion Question

Which treatment is better?
  A) Treatment A for all cases.
  B) Treatment B for all cases.
  C) Treatment A for small stones and B for large stones.
  D) Treatment A for large stones and B for small stones.
To answer, go to `menti.com` and enter 90 74 79.

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Simpson's Paradox occurs when an association between two variables exists when the data is divided into subgroups, but reverses or disappears when the groups are combined.

▶ See more in DSC 80.

Summary, next time

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► The probability that events *A* and *B* both happen is
  $P(A \cap B) = P(A)P(B|A)$.

  ► $P(B|A)$ is the probability that *B* happens given that
    you know *A* happened.

  ► Through re-arranging, we see that $P(B|A) = \frac{P(A \cap B)}{P(B)}$.