DSC 40A

Theoretical Foundations of Data Science I

In This Video

- Many probability questions can be solved by counting, or combinatorics.
- We'll learn how to count sequences and sets.

Sequences vs. Sets

Sequences	Sets
Order matters	Order does not matter
Repetitions allowed	No repetitions allowed
Elements listed in order	Elements listed in no particular order within curly braces
Ex: $2, 4, 5 \neq 4, 2, 5$	Ex: $\{2, 4, 5\} = \{4, 2, 5\}$
Ex: $2, 2, 2 \neq 2, 2$	Ex: $\{2, 2, 2\} = \{2, 2\} = \{2\}$
Ex: $1, 3, 4 = 1, 3, 4$	Ex: $\{1, 3, 4\} = \{1, 3, 4\}$

Sequences

Sequences
Order matters
Repetitions allowed
Elements listed in order
Ex: 2, 4, 5 ≠ 4, 2, 5
Ex: 2, 2, 2 ≠ 2, 2
Ex: 1, 3, 4 = 1, 3, 4

A UCSD PID starts with “A” then has 8 digits. How many UCSD PIDs are possible?

A. 8^{10}

B. 10^{8}

C. 8!

D. 10*9*8*7*6*5*4*3

Sequences

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Ex: 2, 2, 2 ≠ 2, 2
Ex: 1, 3, 4 = 1, 3, 4

How many ways to select a president, vice president, and secretary from a group of 8 people?

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How many ways to select a president, vice president, and secretary from a group of 8 people?

Sequences where repetitions are not allowed are _____.

Sets

There are 24 ice cream flavors. How many ways can you pick 2 different flavors?

A. 24
C. 24*24
B. 24*23
D. 12*23

Sets
Order does not matter
No repetitions allowed
Elements listed in no particular order within curly braces
Ex: {2, 4, 5} = {4, 2, 5}
Ex: {2, 2, 2} = {2, 2} = {2}
Ex: {1, 3, 4} = {1, 3, 4}

Sets

How many ways to select a committee of 3 from a group of 8?

Sets
Order does not matter
No repetitions allowed
Elements listed in no particular order within curly braces
Ex: $\{2, 4, 5\} = \{4, 2, 5\}$
Ex: $\{2, 2, 2\} = \{2, 2\} = \{2\}$
Ex: $\{1, 3, 4\} = \{1, 3, 4\}$

Permutations vs. Combinations

Permutations	Combinations
Order matters	Order does not matter
No repetitions allowed	No repetitions allowed
Counts the number of **sequences of k distinct elements** chosen from n possible elements	Counts the number of **sets of size k** chosen from n possible elements
$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}$	$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$
How many ways to select a president, vice president, and secretary from a group of 8 people? P(8,3)	How many ways to select a committee of 3 from a group of 8? C(8,3)

Sampling Without Replacement

Example 6. There are 20 students in a class. A computer program selects a random sample of students by drawing 5 students at random **without replacement**. What is the chance that a particular student is among the 5 selected students?

from Theory Meets Data by Ani Adhikari, Chapter 4

Sampling Without Replacement

Part 1. Denominator. If you draw a sample of size 5 at random without replacement from a population of size 20, how many different **sets** of individuals could you draw?

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Sampling Without Replacement

Part 2. Numerator. If you draw a sample of size 5 at random without replacement from a population of size 20, how many different **sets** of individuals include a particular person?

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Sampling Without Replacement

Using the complement. If you draw a sample of size 5 at random without replacement from a population of size 20, how many different **sets** of individuals **do not** include a particular person?

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Sampling Without Replacement

Example 6. There are 20 students in a class. A computer program selects a random sample of students by drawing 5 students at random **without replacement**. What is the chance that a particular student is among the 5 selected students?

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Summary

- Sequences vs. sets
- When elements are distinct: permutations vs. combinations

$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

- **Next time:** more examples