DSC 40A

Theoretical Foundations of Data Science I

In This Video

- More examples of using combinatorics to solve probability questions.

Example 7. What is the probability that a randomly generated bitstring of length 10 contains an equal number of zeros and ones?

Example 8. What is the probability that a randomly generated bitstring of length 10 is the string 0011001101?

$$P(n,k) = (n)(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Counting as a Tool for Probability

Example 9. What is the probability that a fair coin flipped 10 times turns up an equal number of heads and tails?

Example 10. What is the probability that a fair coin flipped 10 times turns up HHTTHHTTHT?

$$P(n,k) = (n)(n-1)\ldots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Example 11. What is the probability that a biased coin with $Prob(H) = \frac{1}{3}$ flipped 10 times turns up an equal number of heads and tails?

Example 12. What is the probability that a biased coin with $Prob(H) = \frac{1}{3}$ flipped 10 times turns up HHTTHHTTHT?

$$P(n, k) = (n)(n-1) \ldots (n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n, k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

The Easy Way

Example 6. There are 20 students in a class. A computer program selects a random sample of students by drawing 5 students at random **without replacement**. What is the chance that a particular student is among the 5 selected students?

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Another way to think of sampling without replacement:
1. randomly shuffle all 20 students
2. take the first 5

Practice Problems

Example 13. You were one of 238 individuals who reported for jury duty. If 54 of these people will be assigned to a courtroom, what is the probability that you get assigned to a courtroom?

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How many sets of 54 individuals include you?

      A. C(238, 54)   C. C(238, 53)

      B. C(237, 54)   D. C(237, 53)

Example 14. You were one of 238 individuals who reported for jury duty. Suppose 28 of these individuals are doctors. If 54 of these people will be assigned to a courtroom, what is the probability that exactly 5 doctors get assigned to a courtroom?

Example 15. What is the probability that your five-card poker hand is a straight?

Example 16. Suppose you look at your first card as it is dealt, and you see that it is a Queen. What is the probability that your five-card hand is a straight?

Summary

- Counting is a useful tool for probability.

- Sometimes there's an easy way!

- **Next time:** Bayes Theorem