DSC 40A

Theoretical Foundations of Data Science I

In This Video

 More examples of using combinatorics to solve probability questions.

Counting as a Tool for Probability

Example 7. What is the probability that a randomly generated bitstring of length 10 contains an equal number of zeros and ones?

five 0's five 1's , 10 positions

five positions for 1's : $\{3,7,4,8,2\} \longleftrightarrow 0111001100$

$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Counting as a Tool for Probability

Example 9. What is the probability that a fair coin flipped 10 times turns up an equal number of heads and tails?

Set of fortions for fortions for fortions fortions

Example 10. What is the probability that a fair coin flipped 10 times turns up HHTTHHTTHT?

$$\frac{1}{2^{10}} = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \cdot ... \times \frac{1}{2} = \left(\frac{1}{2}\right)^{10}$$

$$P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}$$

$$C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$$

Counting as a Tool for Probability

Example 11. What is the probability that a biased coin with $Prob(H) = \frac{1}{3}$ flipped He times turns up an equal number of heads and tails?

S= (01) toss seguences of length 10
$$P(E) = \sum_{s \in E} p(s) = \binom{t}{s} + \binom{t}{s} \binom{t}{s}$$

$$E = coin toss seguences of length 10
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$$E = coin toss seguences of length 1$$$$$$$$

Example 12. What is the probability that a biased coin with $Prob(H) = \frac{1}{3}$ flipped 10 times turns up HHTTHHTTHT?

times turns up HHTTHHTTHT?

$$(\frac{1}{3}) * (\frac{1}{3}) * (\frac{2}{3}) *$$

 $P(n,k) = (n)(n-1)\dots(n-k+1) = \frac{n!}{(n-k)!}$ $C(n,k) = \binom{n}{k} = \frac{n!}{k!(n-k)!}$

The Easy Way

Example 6. There are 20 students in a class. A computer program selects a random sample of students by drawing 5 students at random **without replacement**. What is the chance that a particular student is among the 5 selected students?

The Easy Way

Example 6. There are 20 students in a class. A computer program selects a random sample of students by drawing 5 students at random without replacement. What is the chance that a particular student is among the 5 selected students?

Another way to think of sampling without replacement:

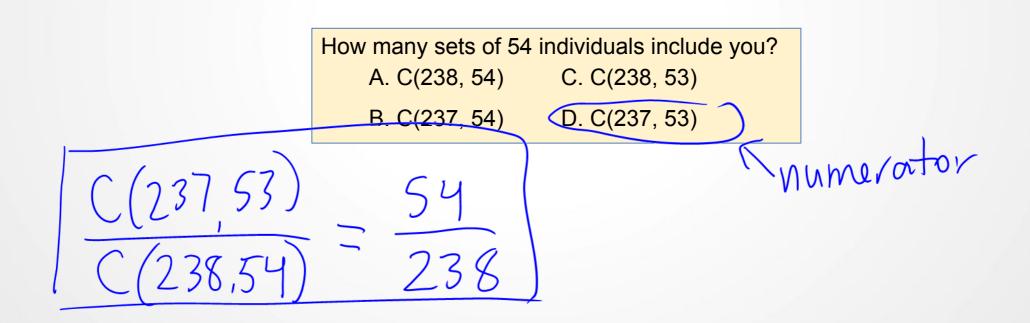
- 1. randomly shuffle all 20 students
- 2. take the first 5

from Theory Meets Data by Ani Achikari, Chapter 4

Example 13. You were one of 238 individuals who reported for jury duty. If 54 of these people will be assigned to a courtroom, what is the probability that you get assigned to a courtroom?

another way: S = sets of s4, chosen from 238 $C(238,54) \leftarrow denominator$

Example 13. You were one of 238 individuals who reported for jury duty. If 54 of these people will be assigned to a courtroom, what is the probability that you get assigned to a courtroom?



Example 14. You were one of 238 individuals who reported for jury duty. Suppose 28 of these individuals are doctors. If 54 of these people will be assigned to a courtroom, what is the probability that exactly 5 doctors get assigned to a courtroom?

S = sets of Sy

individuals

chosen from 238

$$P(5 \text{ doctors}) = \# \text{ sets in S with 5 doctors} = \frac{C(28,5) *C(210, 49)}{C(238,54)}$$

Example 15. What is the probability that your five-card poker hand is a straight?

A, 2, 3, 4, 5, 6, 7, 8, 9, 10, J. Q, K, A

Example 16. Suppose you look at your first card as it is dealt, and you see that it is a Queen. What is the probability that your five card hand is a straight?

Queen. What is the probability that your five-card hand is a straight?

S = possible sets of 4 cards that can be dealt (among 51)

prob(straight) = # sets of 4 cards that make straight when paired with Q

sets of 4 cards

which > 6 4 4 6 which

suite

Summary

- Counting is a useful tool for probability.
- Sometimes there's an easy way!
- Next time: Bayes Theorem