DSC 40A

Theoretical Foundations of Data Science I

Last Time

- Recall that A and B are independent if

$$P(A \text{ and } B) = P(A) * P(B)$$

- A and B are conditionally independent given C if

$$P((A \text{ and } B)|C) = P(A|C) * P(B|C)$$

- Given that C occurs, this says that A and B are independent of one another.

In This Video

- Using Bayes' Theorem to solve the classification problem

Classification

- Making predictions based on examples (training data)
- Response variable is categorical
- Categories are called *classes*
- Examples:
  - decide whether patient has kidney disease
  - identify handwritten digits
  - determine whether an avocado is ripe
  - predict whether credit card activity is fraudulent

Example

Color	Ripeness
bright green	unripe
green-black	ripe
purple-black	ripe
green-black	unripe
purple-black	ripe
bright green	unripe
green-black	ripe
purple-black	ripe
green-black	ripe
green-black	unripe
purple-black	ripe

You have a green-black avocado. Based on this data, would you predict that your avocado is ripe or unripe?

Which class would you predict?

A. ripe
B. unripe

Example

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bright green	unripe
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purple-black	ripe
green-black	unripe
purple-black	ripe
bright green	unripe
green-black	ripe
purple-black	ripe
green-black	ripe
green-black	unripe
purple-black	ripe

You have a green-black avocado. Based on this data, would you predict that your avocado is ripe or unripe?

Strategy: Calculate two probabilities:

P(ripe | green-black)

P(unripe | green-black)

Then choose the class according to the **larger** of these two probabilities.

Bayes' Theorem for Classification

Bayes' Theorem gives another strategy for predicting the class given features.

$$P(B|A) = \frac{P(A|B) * P(B)}{P(A)}$$

B = belonging to a certain class
A = having certain features

$$P(\text{class}|\text{features}) = \frac{P(\text{features}|\text{class}) * P(\text{class})}{P(\text{features})}$$

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Can all be estimated from the training data

Avocado Ripeness

Color	Ripeness
bright green	unripe
green-black	ripe
purple-black	ripe
green-black	unripe
purple-black	ripe
bright green	unripe
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You have a green-black avocado. Based on this data, would you predict that your avocado is ripe or unripe?

$$P(\text{class}|\text{features}) = \frac{P(\text{features}|\text{class}) * P(\text{class})}{P(\text{features})}$$

Shortcut: Both probabilities have same denominator. To find larger one, choose one with larger numerator.

P(ripe | green-black)

P(unripe | green-black)

More Features

Color	Softness	Variety	Ripeness
bright green	firm	Zutano	unripe
green-black	medium	Hass	ripe
purple-black	firm	Hass	ripe
green-black	medium	Hass	unripe
purple-black	soft	Hass	ripe
bright green	firm	Zutano	unripe
green-black	soft	Zutano	ripe
purple-black	soft	Hass	ripe
green-black	soft	Zutano	ripe
green-black	firm	Hass	unripe
purple-black	medium	Hass	ripe

You have a firm green-black Zutano avocado. Based on this data, would you predict that your avocado is ripe or unripe?

Avocado Ripeness

Color	Softness	Variety	Ripeness
bright green	firm	Zutano	unripe
green-black	medium	Hass	ripe
purple-black	firm	Hass	ripe
green-black	medium	Hass	unripe
purple-black	soft	Hass	ripe
bright green	firm	Zutano	unripe
green-black	soft	Zutano	ripe
purple-black	soft	Hass	ripe
green-black	soft	Zutano	ripe
green-black	firm	Hass	unripe
purple-black	medium	Hass	ripe

You have a firm green-black Zutano avocado. Based on this data, would you predict that your avocado is ripe or unripe?

Strategy: Calculate two probabilities:

P(ripe | firm, green-black, Zutano)

P(unripe | firm, green-black, Zutano)

Then choose the class according to the **larger** of these two probabilities.

Avocado Ripeness

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purple-black	soft	Hass	ripe
green-black	soft	Zutano	ripe
green-black	firm	Hass	unripe
purple-black	medium	Hass	ripe

You have a firm green-black Zutano avocado. Based on this data, would you predict that your avocado is ripe or unripe?

Problem: We have not seen an avocado with all these features. Both probabilities will be undefined.

P(ripe | firm, green-black, Zutano)

P(unripe | firm, green-black, Zutano)

Avocado Ripeness

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You have a firm green-black Zutano avocado. Based on this data, would you predict that your avocado is ripe or unripe?

$$P(\text{class}|\text{features}) = \frac{P(\text{features}|\text{class}) * P(\text{class})}{P(\text{features})}$$

Solution: Use Bayes' Theorem, plus a simplifying assumption, to calculate the two numerators.

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$$P(\text{class}|\text{features}) = \frac{P(\text{features}|\text{class}) * P(\text{class})}{P(\text{features})}$$

Simplifying assumption: Within a given class, the features are independent.

P(firm, green-black, Zutano | ripe) =
P(firm | ripe)*P(green-black | ripe)*P(Zutano | ripe)

Conditional Independence

- Recall that A and B are independent if

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green-black	soft	Zutano	ripe
green-black	firm	Hass	unripe
purple-black	medium	Hass	ripe

You have a firm green-black Zutano avocado. Based on this data, would you predict that your avocado is ripe or unripe?

Assuming conditional independence of features given the class, calculate P(firm, green-black, Zutano | unripe).

A. 0
B. 1/4
C. 3/16
D. 1 - (1/7*3/7*2/7)

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Naive Bayes Algorithm

- Bayes' Theorem shows how to calculate P(class | features).

$$P(\text{class}|\text{features}) = \frac{P(\text{features}|\text{class}) * P(\text{class})}{P(\text{features})}$$

- Rewrite the numerator, using the naive assumption of conditional independence of features given the class.
- Estimate each term in the numerator based on the training data.
- Select class based on whichever has the larger numerator.

Summary

- The Naive Bayes algorithm gives a strategy for classifying data according to its features.
- It relies on an assumption of conditional independence of the features.
- **Next time:** application to text classification