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33 points

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Q1

6 points

⚙ Rubı

1. (6 points) Define the extreme mean (EM) of a dataset to be the average of its largest and smallest values. Let

$$f(x) = -3x + 4.$$

Show that for any dataset $x_1 \leq x_2 \leq \cdots \leq x_n$,

$$EM(f(x_1), f(x_2), \ldots, f(x_n)) = f(EM(x_1, x_2, \ldots, x_n)).$$

2

1 +6.0

Fully correct

2 +1.0

Say that the smallest of $f(x_1), f(x_2), \ldots, f(x_n)$ is

and the largest is $f(x_1)$

3 +1.0

Justifying the claim above

(can prove $a < b \Rightarrow f(a) > f(b)$ or

say that order gets reversed by this tranformation)

4 +1.0

Correctly express $f(EM(x_1, x_2, \ldots, x_n)) = f($

5 +1.0

Correctly express

$EM(f(x_1), f(x_2), \ldots, f(x_n)) = \frac{f(x_1)+f(x_n)}{2}$

6 +2.0

Correctly show the equivalence of $f(\frac{x_1+x_n}{2})$ and $\frac{f(x_1)\text{-}}{}$

7 +1.0

Partial credit for showing equivalence

8 +0.0

Incorrect or omitted

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Q2

10 points

Q2.1

4 points

⚙ Rubı

2. (10 points) Consider a new loss function,

$$L(h, y) = e^{(h-y)^2}.$$

Given a dataset $y_1, y_2, \ldots, y_n$, let $R(h)$ represent the empirical risk for the dataset using this loss function.

a) (4 points) For the dataset $\{1, 3, 4\}$, calculate $R(2)$. Simplify your answer as much as possible without a calculator.

3

1 +4.0

Fully correct: $R(2) = \frac{2e+e^4}{3}$

2 +1.0

Correctly express $R(h) = \frac{L(h,y_1)+L(h,y_2)+L(h,y_3)}{3}$

3 +0.5

Partial credit for rubric item [2]: forgot to divide by 3

4 +1.0

First term in numerator calculated correctly: $L(2, 1) = e$

5 +1.0

Second term in numerator calculated correctly: $L(2, 3) = e$

6 +1.0

Third term in numerator calculated correctly: $L(2, 4) = e^4$

7 +2.0

Partial credit for rubric items [4], [5], [6]: small misunderstanding in how to calculate terms (ex. calculating $R'(2)$ instead, not squaring the exponer

8 +0.0

Incorrect or omitted

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Q2.2

6 points

⚙ Rubı

b) (6 points) For the same dataset $\{1, 3, 4\}$, perform one iteration of gradient descent on $R(h)$, starting at an initial

1 +6.0

prediction of $h_0 = 2$ with a step size of $\alpha = \frac{1}{2}$. Show your work and simplify your answer.

Fully correct

2 +1.0

Correct gradient descent update rule: $h_1 = h_0 - \alpha R'(h_0)$

3 +2.0

Correctly calculate derivative using chain rule

$$R'(h) = \frac{1}{n}\sum_{i=1}^{n} e^{(h-y_i)^2} * 2(h - y_i)$$

4 +1.0

Partial credit for derivative (ex. forgetting the 2)

5 +2.0

Correctly calculate terms of derivative

$$R'(2) = \frac{e*(2)+e*(-2)+e^4*(-4)}{3} = -\frac{4e^4}{3}$$

6 +1.5

Partial credit for rubric item [4]: small arithmetic error

7 +1.0

Correctly simplify final answer: $h_1 = 2 + \frac{2}{3}e^4$ or equivalent

8 +0.0

Incorrect or omitted

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Q3

8 points

Rubı

3. (8 points) Suppose you have a dataset

$$\{(x_1, y_1), (x_2, y_2), \ldots, (x_8, y_8)\}$$

with $n = 8$ ordered pairs such that the variance of $\{x_1, x_2, \ldots, x_8\}$ is 50. Let m be the slope of the regression line fit to this data.

Suppose now we fit a regression line to the dataset

$$\{(x_1, y_2), (x_2, y_1), \ldots, (x_8, y_8)\}$$

where the first two y-values have been swapped. Let m' be the slope of this new regression line.

If $x_1 = 3$, $y_1 = 7$, $x_2 = 8$, and $y_2 = 2$, what is the difference between the new slope and the old slope? That is, what is $m' - m$? The answer you get should be a number with no variables.

Hint: There are many equivalent formulas for the slope of the regression line. We recommend using the version of the formula without $\overline{y}$.

1 +8.0

Fully correct

2 +2.0

Correct denominator: $n * Var(x) = 8 * 50$

More space on the next page.

3 +1.0

Partial credit for denominator:
ex. forgot the n

4 +2.0

Knew to separate out the first two terms ($i = 1, 2$) in th
numerator

5 +2.0

Correctly express $m' - m$ in terms of the $i = 1, 2$ nu
terms and the denominator

6 +2.0

Simplify to correct answer $\frac{1}{16}$ or equivalent

7 +1.0

Partial credit for simplification: answer has $\bar{x}$ or $\bar{y}$

8 +0.0

Incorrect or omitted

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Q4

9 points

Q4.1

5 points

Rubı

4. (9 points) Consider the dataset shown below.

$x^{(1)}$	$x^{(2)}$	$x^{(3)}$	y
0	6	8	-5
3	4	5	7
5	-1	-3	4
0	2	1	2

a) (5 points) We want to use multiple regression to fit a prediction rule of the form

$$H(x^{(1)}, x^{(2)}, x^{(3)}) = w_0 + w_1 x^{(1)} x^{(3)} + w_2 (x^{(2)} - x^{(3)})^2.$$

Write down the design matrix X and observation vector $\vec{y}$ for this scenario. No justification needed.

1 +4.0

Correct design matrix X

$$\begin{bmatrix} 1 & 0 & 4 \\ 1 & 15 & 1 \\ 1 & -15 & 4 \\ 1 & 0 & 1 \end{bmatrix}$$

2 +1.0

Partial credit: design matrix has 4 rows and 3 columns

3 +1.0

Partial credit: design matrix has a first column of ones

4 +3.0

Partial credit: seems to be creating the design matrix cor
but more than one arithmetic mistake

5 +1.0

Correct observation vector y

$$\begin{bmatrix} -5 \\ 7 \\ 4 \\ 2 \end{bmatrix}$$

6 +0.0

Incorrect or omitted

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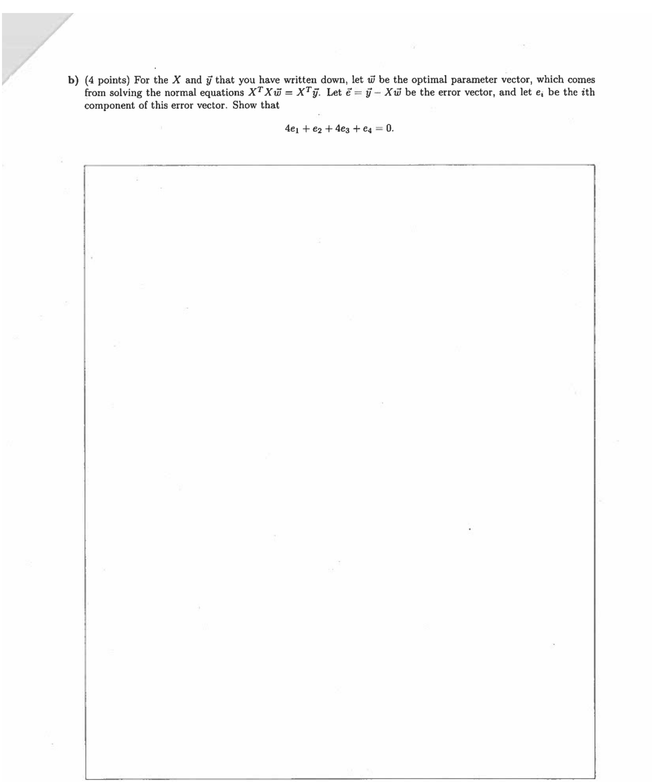
Q4.2

4 points

Rub



b) (4 points) For the X and $\vec{y}$ that you have written down, let $\vec{w}$ be the optimal parameter vector, which comes from solving the normal equations $X^TX\vec{w} = X^T\vec{y}$. Let $\vec{e} = \vec{y} - X\vec{w}$ be the error vector, and let e_i be the ith component of this error vector. Show that

$$4e_1 + e_2 + 4e_3 + e_4 = 0.$$

1 +4.0

Strategy 1: State that
$\vec{e}$ is orthogonal to the columns of X and
use the column with entries 4, 1, 4, 1 to conclude the res

2 +2.0

Strategy 1: State that
$\vec{e}$ is orthogonal to the columns of X or
$X^T\vec{e} = 0$

3 +4.0

Strategy 2: Calcuate $\vec{w}$ by solving normal equations
$X^TX\vec{w} = X^T\vec{y}$, then calculating error vector
$\vec{e} = \vec{y} - X\vec{w}$, resulting in correct error vector

$$\begin{bmatrix} -3.5 \\ 3.5 \\ 3.5 \end{bmatrix}$$

4 +2.0

Strategy 2: Correct approach, but incorrect error vector

5 +0.0

Incorrect or omitted

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