DSC 40A - Homework 6

Due: Tuesday, May 23 at 11:59pm

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. Homeworks are due to Gradescope by 11:59pm on the due date. You can use a slip day to extend the deadline by 24 hours.

Homework will be evaluated not only on the correctness of your answers, but on your ability to present your ideas clearly and logically. You should **always explain and justify** your conclusions, using sound reasoning. Your goal should be to convince the reader of your assertions. If a question does not require explanation, it will be explicitly stated.

Homeworks should be written up and turned in by each student individually. You may talk to other students in the class about the problems and discuss solution strategies, but you should not share any written communication and you should not check answers with classmates. You can tell someone how to do a homework problem, but you cannot show them how to do it.

For each problem you submit, you should **cite your sources** by including a list of names of other students with whom you discussed the problem. Instructors do not need to be cited.

This homework will be graded out of 50 points. The point value of each problem or sub-problem is indicated by the number of avocados shown.

Notes:

- For full credit, make sure to **assign pages to questions** when you upload your submission to Gradescope.
- For Problem 5 part (c), code your answer in the supplementary Jupyter notebook (linked). You'll need to turn in your completed Python file to Gradescope separately from the rest of this homework, in a file called `hw6code.py`. This part will be autograded, so no explanation is needed.
- Throughout this homework, please **give your answers in two forms**:
  - unsimplified, in terms of factorials, exponents, the permutation formula $P(n, k)$, and the combination formula $C(n, k) = \binom{n}{k}$, and
  - as an integer, a decimal approximation to three decimal places, or in scientific notation.

Problem 0. Reflection and Feedback Form

Make sure to fill out this Reflection and Feedback Form, linked here for three points on this homework! This form is primarily for your benefit; research shows that reflecting and summarizing knowledge helps you understand and remember it.

Problem 1. Baseball

a) Suppose that your baseball team has 12 players but you only have 8 matching team hats. How many ways are there for you to select who gets a hat and who doesn't?

b) Suppose that your baseball team has 12 players and you have 16 jerseys, numbered 1 through 16. You will give out one jersey to each player, and four will be leftover. How many ways are there for you to assign jerseys to players?

c) As before, your baseball team has 12 players and you have 16 jerseys, numbered 1 through 16. You will give out one jersey to each player, and four will be leftover. How many ways are there for you to assign jerseys to players if you must give out jersey number 7?

d) Suppose that at the batting cage, where a machine pitches the ball, each time you swing the bat, your probability of hitting the ball is 3/5. If you swing 9 times, what is the probability that you hit the ball exactly 7 times?

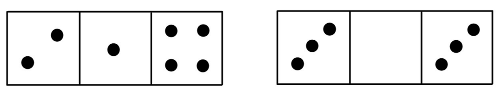
e) Suppose that when a pitcher is throwing the ball, your probability of hitting the ball depends on the kind of pitch. Your probability of hitting the ball is

- $\frac{1}{2}$ for a fastball,
- $\frac{1}{3}$ for a breaking ball, and
- $\frac{1}{4}$ for a changeup

Suppose that at practice, a pitcher throws you three fastballs, three breaking balls, and three changeups. What is the probability that you miss one breaking ball and one changeup, but hit all 7 other balls?

Problem 2. From Dominoes to Trominoes

A *tromino* is a rectangular tile divided into three sections. On each section, there is some number of dots between 0 and 6, inclusive. Two example trominoes are shown below.



A complete set of trominoes consists of every possible combination of dots on each section.

a) Draw a picture of all the trominoes that have at least one 1, at least one 3, and only 1s and 3s (no other numbers). No explanation needed.

b) Draw a picture of all the trominoes that have a 1, a 3, and a 6. No explanation needed.

c) How many trominoes are in a complete set?

Hint: The answers to parts (a) and (b) should help you.

d) In class, we calculated that the number of dominoes in a complete set is 28. How does your answer to part (c) relate to the number 28? Explain why this makes sense.

Problem 3. 5 Surprise Mini Brands

A toy called *5 Surprise Mini Brands* is a plastic capsule containing five miniature replicas of branded household products, like Skippy Peanut Butter, Dove Bodywash, Kikkoman Soy Sauce, or Breyers Ice Cream. Each capsule is a surprise; you don't know which minis it will contain until you open it! We'll assume for this problem that there are 70 possible products, each of which is manufactured in equal quantities, so you're no more likely to get any one product than any other.

a) Suppose that each capsule's contents are selected uniformly at random **without replacement** from among the 70 possibilities. If you buy two capsules, what is the probability that you end up with exactly five distinct products overall?

b) Suppose that each capsule's contents are selected uniformly at random **without replacement** from among the 70 possibilities. If you buy two capsules, what is the probability that you end up with exactly ten distinct products overall?

c) Suppose that each capsule's contents are selected uniformly at random **with replacement** from among the 70 possibilities. If you buy two capsules, what is the probability that you end up with exactly ten distinct products overall?

d) Suppose that each capsule's contents are selected uniformly at random **with replacement** from among the 70 possibilities. If you buy two capsules, what is the probability that you end up with exactly nine distinct products overall?

Problem 4. Book Club

A book club includes p people, and there are b books that the book club is considering reading this month. Before deciding on which book to read this month, the book club president asks each person which of the b books they have already read.

A *book club description* is a description of who, among p people, has already read each of b books. For example, if a book club has $p = 3$ people (Ben, Tunan, Pallavi) and there are $b = 2$ books (Book 1, Book 2), one possible book club description is as follows:

- Ben has read Book 1.
- Tunan has read Book 1 and Book 2.
- Pallavi has read neither Book 1 nor Book 2.

a) How many book club descriptions are possible? Give your answer as a formula involving p and b, with explanation of where the formula comes from.

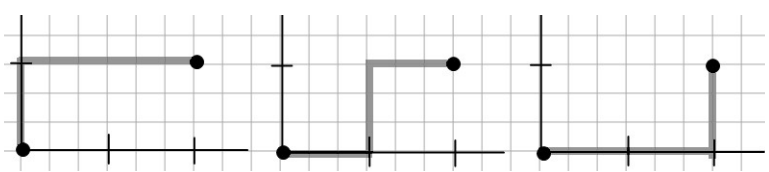
b) How many book club descriptions are such that nobody has read Book 1? Give your answer as a formula involving p and b, with explanation of where the formula comes from.

c) How many book club descriptions are such that there is at least one book that nobody has read? Give your answer as a formula involving p and b, with explanation of where the formula comes from.

Problem 5. Paths

Suppose we are given two nonnegative integers n and m. We want to find all the different paths from $(0, 0)$ to (n, m) that use only upward and rightward steps of length one.

For example, if $n = 2$ and $m = 1$, the goal is to find all paths from the origin to $(2, 1)$. There are three such paths, as shown below.



We can represent each path as a sequence of up and right arrows. In the example above, these sequences would be $\uparrow\rightarrow\rightarrow$ for the leftmost path, $\rightarrow\uparrow\rightarrow$ for the middle path, and $\rightarrow\rightarrow\uparrow$ for the rightmost path.

a) Find all paths from the origin to $(3, 2)$. You can either draw them out or describe them using up and right arrows. No explanation needed.

b) For nonnegative integers n and m, let $P(n,m)$ be the number of paths from $(0,0)$ to (n,m) using upward and rightward steps of length 1. Explain why $P(n,m)$ satisfies each of the following:

$$P(0,m) = 1 \tag{1}$$
$$P(n,0) = 1 \tag{2}$$
$$P(n,m) = P(n-1,m) + P(n,m-1) \quad \text{for } n > 0 \text{ and } m > 0. \tag{3}$$

c) In the supplementary Jupyter notebook (linked), fill in the blanks (marked by ...) in the provided partially complete `paths` function. This function should take as input two nonnegative integers n and m return an array containing all the different paths from $(0,0)$ to (n,m) that use only upward and rightward steps of length one.

d) For nonnegative integers n and m, how many paths are there from $(0,0)$ to (n,m) using upward and rightward steps of length 1? Give your answer as a formula involving n and m, and justify your answer.