

Lecture 22 – Independence and Conditional Independence



DSC 40A, Spring 2023

Announcements

- ▶ Discussion section is tonight at 7pm and 8pm in FAH 1101. Tonight's assignment is the last groupwork assignment!
- ▶ Great source of practice problems for recent content: stat88.org/textbook.
- ▶ Also check out the Probability Roadmap on the [resources tab of the course website](#).
- ▶ Consider applying for the [HDSI Undergrad Scholarship Program](#)!

Agenda

- ▶ Independence.
- ▶ Conditional independence.

Independence

Independent events

- ▶ A and B are **independent events** if one event occurring does not affect the chance of the other event occurring.
- ▶ To check if A and B are independent, use whichever is easiest:
 - ▶ $P(B|A) = P(B)$.
 - ▶ $P(A|B) = P(A)$.
 - ▶ $P(A \cap B) = P(A) \cdot P(B)$.

Example: cards

♥: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♦: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♣: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♠: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

- ▶ Suppose you draw two cards, one at a time.
 - ▶ A is the event that the first card is a heart.
 - ▶ B is the event that the second card is a club.
- ▶ If you draw the cards **with** replacement, are A and B independent?
- ▶ If you draw the cards **without** replacement, are A and B independent?

Example: cards

♥: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♦: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♣: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♠: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

- ▶ Suppose you draw one card from a deck of 52.
 - ▶ A is the event that the card is a heart.
 - ▶ B is the event that the card is a face card (J, Q, K).
- ▶ Are A and B independent?

Assuming independence

- ▶ Sometimes we assume that events are independent to make calculations easier.
- ▶ Real-world events are almost never exactly independent, but they may be close.

Example: breakfast

1% of UCSD students are DSC majors. 25% of UCSD students eat avocado toast for breakfast. Assuming that being a DSC major and eating avocado toast for breakfast are independent:

1. What percentage of DSC majors eat avocado toast for breakfast?
2. What percentage of UCSD students are DSC majors who eat avocado toast for breakfast?

Conditional independence

Conditional independence

- ▶ Sometimes, events that are dependent *become* independent, upon learning some new information.
- ▶ Or, events that are independent can become dependent, given additional information.

Example: cards

♥: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♦: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♣: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, A

♠: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

- ▶ Your dog ate the King of Clubs. Suppose you draw one card from a deck of 51.
 - ▶ A is the event that the card is a heart.
 - ▶ B is the event that the card is a face card (J, Q, K).
- ▶ Are A and B independent?

Example: cards

♥: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♦: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♣: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, A

♠: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

- ▶ Your dog ate the King of Clubs. Suppose you draw one card from a deck of 51.
 - ▶ A is the event that the card is a heart.
 - ▶ B is the event that the card is a face card (J, Q, K).
- ▶ Suppose you learn that the card is red. Are A and B independent given this new information?

Conditional independence

- ▶ Recall that A and B are independent if

$$P(A \cap B) = P(A) \cdot P(B)$$

- ▶ A and B are **conditionally independent** given C if

$$P((A \cap B)|C) = P(A|C) \cdot P(B|C)$$

- ▶ Given that C occurs, this says that A and B are independent of one another.

Assuming conditional independence

- ▶ Sometimes we assume that events are conditionally independent to make calculations easier.
- ▶ Real-world events are almost never exactly conditionally independent, but they may be close.

Example: Harry Potter and Discord

Suppose that 50% of UCSD students like Harry Potter and 80% of UCSD students use Discord. What is the probability that a random UCSD student likes Harry Potter and uses Discord, assuming that these events are conditionally independent given that a person is a UCSD student?

Independence vs. conditional independence

- ▶ Is it reasonable to assume conditional independence of
 - ▶ liking Harry Potter
 - ▶ using Discordgiven that a person is a UCSD student?
- ▶ Is it reasonable to assume independence of these events in general, among all people?

Discussion Question

Which assumptions do you think are reasonable?

- Both
- Conditional independence only
- Independence (in general) only
- Neither

Independence vs. conditional independence

In general, there is **no relationship** between independence and conditional independence. All four scenarios below are possible.

- ▶ **Scenario 1:** A and B **are** independent. A and B **are** conditionally independent given C.
- ▶ **Scenario 2:** A and B **are** independent. A and B **are not** conditionally independent given C.
- ▶ **Scenario 3:** A and B **are not** independent. A and B **are** conditionally independent given C.
- ▶ **Scenario 4:** A and B **are not** independent. A and B **are not** conditionally independent given C.

Example: constructing events

- ▶ Consider a sample space $S = \{1, 2, 3, 4, 5, 6\}$ where all outcomes are equally likely.
- ▶ For each scenario, specify events A , B , and C that satisfy the given conditions. (e.g. $A = \{2, 5, 6\}$)
- ▶ Choose events that are neither impossible nor certain, i.e. $0 < P(A), P(B), P(C) < 1$.

Scenario 1: A and B **are** independent. A and B **are** conditionally independent given C .

Example: constructing events

- ▶ Consider a sample space $S = \{1, 2, 3, 4, 5, 6\}$ where all outcomes are equally likely.
- ▶ For each scenario, specify events A , B , and C that satisfy the given conditions. (e.g. $A = \{2, 5, 6\}$)
- ▶ Choose events that are neither impossible nor certain, i.e. $0 < P(A), P(B), P(C) < 1$.

Scenario 2: A and B **are** independent. A and B **are not** conditionally independent given C .

Example: constructing events

- ▶ Consider a sample space $S = \{1, 2, 3, 4, 5, 6\}$ where all outcomes are equally likely.
- ▶ For each scenario, specify events A , B , and C that satisfy the given conditions. (e.g. $A = \{2, 5, 6\}$)
- ▶ Choose events that are neither impossible nor certain, i.e. $0 < P(A), P(B), P(C) < 1$.

Scenario 3: A and B **are not** independent. A and B **are** conditionally independent given C .

Example: constructing events

- ▶ Consider a sample space $S = \{1, 2, 3, 4, 5, 6\}$ where all outcomes are equally likely.
- ▶ For each scenario, specify events A , B , and C that satisfy the given conditions. (e.g. $A = \{2, 5, 6\}$)
- ▶ Choose events that are neither impossible nor certain, i.e. $0 < P(A), P(B), P(C) < 1$.

Scenario 4: A and B **are not** independent. A and B **are not** conditionally independent given C .

Summary

Summary

- ▶ Two events A and B are **independent** when knowledge of one event does not change the probability of the other event.
 - ▶ Equivalent conditions: $P(B|A) = P(B)$, $P(A|B) = P(A)$, $P(A \cap B) = P(A) \cdot P(B)$.
- ▶ Two events A and B are **conditionally independent** if they are independent given knowledge of a third event, C .
 - ▶ Condition: $P((A \cap B)|C) = P(A|C) \cdot P(B|C)$.
- ▶ In general, there is no relationship between independence and conditional independence.
- ▶ **Next time:** Using Bayes' theorem and conditional independence to solve the **classification problem** in machine learning.