

# Module 17 - Sequences, Permutations, and Combinations



DSC 40A, Summer 2023

# Agenda

- ▶ Sequences, permutations, and combinations.

# Motivation

- ▶ Many problems in probability involve counting.
  - ▶ Suppose I flip a fair coin 100 times. What's the probability I see 34 heads?
  - ▶ Suppose I draw 3 cards from a 52 card deck. What's the probability they all are all from the same suit?
- ▶ In order to solve such problems, we first need to learn how to count.
- ▶ The area of math that deals with counting is called **combinatorics**.

## Selecting elements (i.e. sampling)

- ▶ Many experiments involve choosing  $k$  elements randomly from a group of  $n$  possible elements. This group is called a **population**.
  - ▶ If drawing cards from a deck, the population is the deck of all cards.
  - ▶ If selecting people from DSC 40A, the population is everyone in DSC 40A.
- ▶ Two decisions:
  - ▶ Do we select elements with or without **replacement**?
  - ▶ Does the **order** in which things are selected matter?

## Sequences

- ▶ A **sequence** of length  $k$  is obtained by selecting  $k$  elements from a group of  $n$  possible elements **with replacement**, such that **order matters**.
- ▶ **Example:** Draw a card (from a standard 52-card deck), put it back in the deck, and repeat 4 times.
- ▶ **Example:** A UCSD PID starts with “A” then has 8 digits. How many UCSD PIDs are possible?

# Sequences

In general, the number of ways to select  $k$  elements from a group of  $n$  possible elements such that **repetition is allowed** and **order matters** is  $n^k$ .

(Note: We mentioned this fact in the lecture on clustering!)

# Permutations

- ▶ A **permutation** is obtained by selecting  $k$  elements from a group of  $n$  possible elements **without replacement**, such that **order matters**.
- ▶ **Example:** Draw 4 cards (without replacement) from a standard 52-card deck.
- ▶ **Example:** How many ways are there to select a president, vice president, and secretary from a group of 8 people?

# Permutations

- ▶ In general, the number of ways to select  $k$  elements from a group of  $n$  possible elements such that **repetition is not allowed** and **order matters** is

$$P(n, k) = (n)(n - 1)\dots(n - k + 1)$$

- ▶ To simplify: recall that the definition of  $n!$  is

$$n! = (n)(n - 1)\dots(2)(1)$$

- ▶ Given this, we can write

$$P(n, k) = \frac{n!}{(n - k)!}$$



## Discussion Question

UCSD has 7 colleges. How many ways can I rank my top 3 choices?

- a) 21
- b) 210
- c) 343
- d) 2187
- e) None of the above.

## Special case of permutations

- ▶ Suppose we have  $n$  people. The total number of ways I can rearrange these  $n$  people in a line is
  
  
  
  
  
  
  
  
  
  
- ▶ This is consistent with the formula

$$P(n, n) = \frac{n!}{(n - n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

# Combinations

- ▶ A **combination** is a set of  $k$  items selected from a group of  $n$  possible elements **without replacement**, such that **order does not matter**.
- ▶ **Example:** There are 24 ice cream flavors. How many ways can you pick two flavors?





## From permutations to combinations

- ▶ There is a close connection between:
  - ▶ the number of permutations of  $k$  elements selected from a group of  $n$ , and
  - ▶ the number of combinations of  $k$  elements selected from a group of  $n$

$$\# \text{ combinations} = \frac{\# \text{ permutations}}{\# \text{ orderings of } k \text{ items}}$$

- ▶ Since  $\# \text{ permutations} = \frac{n!}{(n-k)!}$  and  $\# \text{ orderings of } k \text{ items} = k!$ , we have

$$C(n, k) = \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

# Combinations

In general, the number of ways to select  $k$  elements from a group of  $n$  elements such that **repetition is not allowed** and **order does not matter** is

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The symbol  $\binom{n}{k}$  is pronounced “ $n$  choose  $k$ ”, and is also known as the **binomial coefficient**.

## Example: committees

- ▶ How many ways are there to select a president, vice president, and secretary from a group of 8 people?
  
- ▶ How many ways are there to select a committee of 3 people from a group of 8 people?
  
- ▶ If you're ever confused about the difference between permutations and combinations, **come back to this example.**



## Summary

## Summary

- ▶ A **sequence** is obtained by selecting  $k$  elements from a group of  $n$  possible elements with replacement, such that order matters.
  - ▶ Number of sequences:  $n^k$ .
- ▶ A **permutation** is obtained by selecting  $k$  elements from a group of  $n$  possible elements without replacement, such that order matters.
  - ▶ Number of permutations:  $P(n, k) = \frac{n!}{(n-k)!}$ .
- ▶ A **combination** is obtained by selecting  $k$  elements from a group of  $n$  possible elements without replacement, such that order does not matter.
  - ▶ Number of combinations:  $\binom{n}{k} = \frac{n!}{(n-k)!k!}$ .