

Module 19 - More Probability and Combinatorics Examples



DSC 40A, Summer 2023

Agenda

- ▶ Lots of examples.

Last time

Last time we answered the same question using several different techniques.

Question 1: There are 20 students in a class. Avi is one of them. Suppose we select 5 students in the class uniformly at random **without replacement**. What is the probability that Avi is among the 5 selected students?

With vs. without replacement

Discussion Question

We've determined that a probability that a random sample of 5 students from a class of 20 **without replacement** contains Avi (one student in particular) is $\frac{1}{4}$.

Suppose we instead sampled **with replacement**. Would the resulting probability be equal to, greater than, or less than $\frac{1}{4}$?

- a) Equal to
- b) Greater than
- c) Less than

Art supplies

Question 2, Part 1: We have 12 art supplies: 5 markers and 7 crayons. In how many ways can we select 4 art supplies?

Art supplies

Question 2, Part 2: We have 12 art supplies: 5 markers and 7 crayons. In how many ways can we select 4 art supplies such that we have...

1. 2 markers and 2 crayons?
2. 3 markers and 1 crayon?

Art supplies

Question 2, Part 3: We have 12 art supplies: 5 markers and 7 crayons. We randomly select 4 art supplies. What's the probability that we selected at least 2 markers?

Fair coin

Question 3: Suppose we flip a **fair coin** 10 times.

1. What is the probability that we see the specific sequence THTTHTHHTH?
2. What is the probability that we see an equal number of heads and tails?

Unfair coin

Question 4: Suppose we flip an **unfair coin** 10 times. The coin is biased such that for each flip, $P(\text{heads}) = \frac{1}{3}$.

1. What is the probability that we see the specific sequence THTTHTHHTH?
2. What is the probability that we see an equal number of heads and tails?

Deck of cards

- ▶ There are 52 cards in a standard deck (4 suits, 13 values).

♥: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♦: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♣: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

♠: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

- ▶ In poker, each player is dealt 5 cards, called a **hand**. The order of cards in a hand does not matter.

Deck of cards

1. How many 5 card hands are there in poker?
2. How many 5 card hands are there where all cards are of the same suit (a **flush**)?

3. How many 5 card hands are there that include a **four-of-a-kind** (four cards of the same value)?

4. How many 5 card hands are there that have a **straight** (all card values consecutive)?

5. How many 5 card hands are there that are a **straight flush** (all card values consecutive and of the same suit)?

6. How many 5 card hands are there that include exactly **one pair** (values aabcd)?

Summary

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- ▶ A **sequence** is obtained by selecting k elements from a group of n possible elements with replacement, such that order matters.
 - ▶ Number of sequences: n^k .
- ▶ A **permutation** is obtained by selecting k elements from a group of n possible elements without replacement, such that order matters.
 - ▶ Number of permutations: $P(n, k) = \frac{n!}{(n-k)!}$.
- ▶ A **combination** is obtained by selecting k elements from a group of n possible elements without replacement, such that order does not matter.
 - ▶ Number of combinations: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.