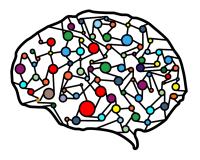# Module 21 – Independence

DSC 40A, Summer 2023

Announcements

▶ Homework 3 is due **tomorrow at 11:59pm**.

▶ Great source of practice problems for recent content:
  stat88.org/textbook.

▶ Also check out the Probability Roadmap on the resources
  tab of the course website.

▶ Reminder: Final will be closed
  books/notes/electronics/web. You will be allowed to
  keep with you two A4-sized sheets (four sides) with any
  content you want.

▶ Extra Credit: If 90% or greater response rate on SET
  evaluations (formerly CAPE) by August 4, class gets a 0.5%
  boost to overall grade.

Agenda

- ▶ Recap of Module 20.

- ▶ Independence.

Last time

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- A set of events $E_1, E_2, ..., E_k$ is a **partition** of S if each outcome in S is in exactly one E_i.
- The **Law of Total Probability** states that if A is an event and $E_1, E_2, ..., E_k$ is a partition of S, then

$$P(A) = P(E_1) \cdot P(A|E_1) + P(E_2) \cdot P(A|E_2) + ... + P(E_k) \cdot P(A|E_k)$$

$$= \sum_{i=1}^{k} P(E_i) \cdot P(A|E_i)$$

- **Bayes' Theorem** states that

$$P(B|A) = \frac{P(B) \cdot P(A|B)}{P(A)}$$

- We often re-write the denominator $P(A)$ in Bayes' Theorem using the Law of Total Probability.

Discussion Question

Consider any two events A and B. Choose the expression that's equivalent to

$$P(B|A) + P(\bar{B}|A).$$

a) $P(A)$
b) $1 - P(B)$
c) $P(B)$
d) $P(\bar{B})$
e) 1

Example: prosecutor's fallacy

A bank was robbed yesterday by one person. Consider the following facts about the crime:

- ▶ The person who robbed the bank wore Nikes.
- ▶ Of the 10,000 other people who came to the bank yesterday, only 10 of them wore Nikes.

The prosecutor finds the prime suspect, and states that "given this evidence, the chance that the prime suspect was not at the crime scene is 1 in 1,000".

1. What is wrong with this statement?

2. Find the probability that the prime suspect is guilty given only the evidence in the exercise.

Independence

Updating probabilities

▶ Bayes' theorem describes how to update the probability of one event, given that another event has occurred.

$$P(B|A) = \frac{P(B) \cdot P(A|B)}{P(A)}$$

  ▶ $P(B)$ can be thought of as the "prior" probability of B occurring, before knowing anything about A.

  ▶ $P(B|A)$ is sometimes called the "posterior" probability of B occurring, given that A occurred.

▶ What if knowing that A occurred doesn't change the probability that B occurs? In other words, what if

$$P(B|A) = P(B)$$

Independent events

▶ *A* and *B* are **independent events** if one event occurring does not affect the chance of the other event occurring.

$$P(B|A) = P(B) \qquad P(A|B) = P(A)$$

▶ Otherwise, *A* and *B* are **dependent events**.

▶ Using Bayes' theorem, we can show that if one of the above statements is true, then so is the other.

Independent events

▶ **Equivalent definition:** *A* and *B* are independent events if

$$P(A \cap B) = P(A) \cdot P(B)$$

▶ To check if *A* and *B* are independent, use whichever is easiest:

  ▶ $P(B|A) = P(B)$.

  ▶ $P(A|B) = P(A)$.

  ▶ $P(A \cap B) = P(A) \cdot P(B)$.

Mutual exclusivity and independence

Discussion Question

Suppose A and B are two events with non-zero probabilities. Is it possible for A and B to be both mutually exclusive and independent?
  a) Yes
  b) No

Example: Venn diagrams

For three events *A*, *B*, and *C*, we know that

▶ *A* and *C* are independent,
▶ *B* and *C* are independent,
▶ *A* and *B* are mutually exclusive,
▶ $P(A \cup C) = \frac{2}{3}$, $P(B \cup C) = \frac{3}{4}$, $P(A \cup B \cup C) = \frac{11}{12}$.

Find $P(A)$, $P(B)$, and $P(C)$.

Summary

Summary

▶ Two events *A* and *B* are **independent** when knowledge of one event does not change the probability of the other event.

▶ There are there equivalent definitions of independence:
  ▶ $P(B|A) = P(B)$

  ▶ $P(A|B) = P(A)$

  ▶ $P(A \cap B) = P(A) \cdot P(B)$