Module 24 – More Naive Bayes

DSC 40A, Summer 2023

Final is Friday from 8:00a-10:59a

- Final will be closed books/notes/electronics/web. You will be allowed to keep with you two A4-sized sheets (four sides) with any content you want.
- Leave all answers **unsimplified** in terms of permutations, combinations, factorials, exponents, etc.
- We will not answer questions during the exam. State your assumptions if anything is unclear.
- Location is slated in regular room, RWAC 0115.

Agenda

- Naive Bayes with smoothing.
- Application — text classification.

Naive Bayes with smoothing

Recap: Naive Bayes classifier

- We want to predict a class, given certain features.
- Using Bayes' theorem, we write

$$P(\text{class}|\text{features}) = \frac{P(\text{class}) \cdot P(\text{features}|\text{class})}{P(\text{features})}$$

- For each class, we compute the numerator using the **naive assumption of conditional independence of features given the class**.
- We estimate each term in the numerator based on the training data.
- We predict the class with the largest numerator.
  - Works if we have multiple classes, too!

Example: avocados

color	softness	variety	ripeness
bright green	firm	Zutano	unripe
green-black	medium	Hass	ripe
purple-black	firm	Hass	ripe
green-black	medium	Hass	unripe
purple-black	soft	Hass	ripe
bright green	firm	Zutano	unripe
green-black	soft	Zutano	ripe
purple-black	soft	Hass	ripe
green-black	soft	Zutano	ripe
green-black	firm	Hass	unripe
purple-black	medium	Hass	ripe

You have a soft green-black Hass avocado. Based on this data, would you predict that your avocado is ripe or unripe?

Uh oh...

- There are no soft unripe avocados in the data set.
- The estimate $P(\text{soft}|\text{unripe}) \approx \frac{\text{\# soft unripe avocados}}{\text{\# unripe avocados}}$ is 0.
- The estimated numerator,
  $P(\text{unripe}) \cdot P(\text{soft, green-black, Hass}|\text{unripe}) = P(\text{unripe}) \cdot P(\text{soft}|\text{unripe}) \cdot P(\text{green-black}|\text{unripe}) \cdot P(\text{Hass}|\text{unripe})$,
  is also 0.
- But just because there isn't a soft unripe avocado in the data set, doesn't mean that it's impossible for one to exist!
- **Idea:** Adjust the numerators and denominators of our estimate so that they're never 0.

Smoothing

- **Without** smoothing:

$$P(\text{soft}|\text{unripe}) \approx \frac{\#\text{ soft unripe}}{\#\text{ soft unripe} + \#\text{ medium unripe} + \#\text{ firm unripe}}$$

$$P(\text{medium}|\text{unripe}) \approx \frac{\#\text{ medium unripe}}{\#\text{ soft unripe} + \#\text{ medium unripe} + \#\text{ firm unripe}}$$

$$P(\text{firm}|\text{unripe}) \approx \frac{\#\text{ firm unripe}}{\#\text{ soft unripe} + \#\text{ medium unripe} + \#\text{ firm unripe}}$$

- **With** smoothing:

$$P(\text{soft}|\text{unripe}) \approx \frac{\#\text{ soft unripe} + 1}{\#\text{ soft unripe} + 1 + \#\text{ medium unripe} + 1 + \#\text{ firm unripe} + 1}$$

$$P(\text{medium}|\text{unripe}) \approx \frac{\#\text{ medium unripe} + 1}{\#\text{ soft unripe} + 1 + \#\text{ medium unripe} + 1 + \#\text{ firm unripe} + 1}$$

$$P(\text{firm}|\text{unripe}) \approx \frac{\#\text{ firm unripe} + 1}{\#\text{ soft unripe} + 1 + \#\text{ medium unripe} + 1 + \#\text{ firm unripe} + 1}$$

- When smoothing, we add 1 to the count of every group whenever we're estimating a conditional probability.

Example: avocados, with smoothing

color	softness	variety	ripeness
bright green	firm	Zutano	unripe
green-black	medium	Hass	ripe
purple-black	firm	Hass	ripe
green-black	medium	Hass	unripe
purple-black	soft	Hass	ripe
bright green	firm	Zutano	unripe
green-black	soft	Zutano	ripe
purple-black	soft	Hass	ripe
green-black	soft	Zutano	ripe
green-black	firm	Hass	unripe
purple-black	medium	Hass	ripe

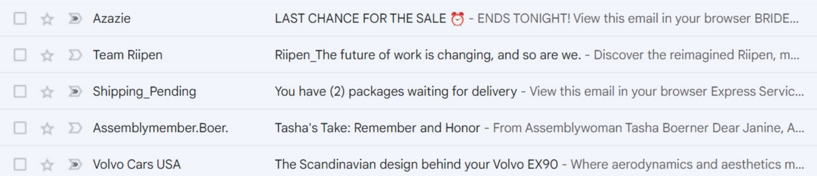
You have a soft green-black Hass avocado. Using Naive Bayes, **with smoothing**, would you predict that your avocado is ripe or unripe?

Text classification

Text classification

- Text classification problems include:
  - Sentiment analysis (e.g. positive and negative customer reviews).
  - Determining genre (news articles, blog posts, etc.).
  - Spam filtering.

Spam filtering



- **Our goal:** given the body of an email, determine whether it's **spam** or **ham** (not spam).
- **Question:** How do we come up with features?

Features

Idea:

- Choose a **dictionary** of d words.

- Represent each email with a **feature vector** $\vec{x}$:

$$\vec{x} = \begin{bmatrix} x^{(1)} \\ x^{(2)} \\ \dots \\ x^{(d)} \end{bmatrix}$$

  where
  - $x^{(i)} = 1$ if word i is present in the email, and
  - $x^{(i)} = 0$ otherwise.

This is called the **bag-of-words** model. This model ignores the frequency and meaning of words.

Concrete example

- Dictionary: “prince”, “money”, “free”, and “just”.

- Dataset of 5 emails (red are spam, green are ham):
  - **“I am the prince of UCSD and I demand money.”**
  - **“Tapioca Express: redeem your free Thai Iced Tea!”**
  - **“DSC 10: free points if you fill out CAPEs!”**
  - **“Click here to make a tax-free donation to the IRS.”**
  - **“Free career night at Prince Street Community Center.”**

Naive Bayes for spam classification

$$P(\text{class} \mid \text{features}) = \frac{P(\text{class}) \cdot P(\text{features} \mid \text{class})}{P(\text{features})}$$

- To classify an email, we'll use Bayes' theorem to calculate the probability of it belonging to each class:
  - $P(\text{spam} \mid \text{features})$.
  - $P(\text{ham} \mid \text{features})$.

- We'll predict the class with a larger probability.

Naive Bayes for spam classification

$$P(\text{class} \mid \text{features}) = \frac{P(\text{class}) \cdot P(\text{features} \mid \text{class})}{P(\text{features})}$$

- Note that the formulas for $P(\text{spam} \mid \text{features})$ and $P(\text{ham} \mid \text{features})$ have the same denominator, $P(\text{features})$.
- Thus, we can find the larger probability just by comparing numerators:
  - $P(\text{spam}) \cdot P(\text{features} \mid \text{spam})$.
  - $P(\text{ham}) \cdot P(\text{features} \mid \text{ham})$.

Naive Bayes for spam classification

Discussion Question

We need to determine four quantities:

1. $P(\text{features} \mid \text{spam})$.
2. $P(\text{features} \mid \text{ham})$.
3. $P(\text{spam})$.
4. $P(\text{ham})$.

Which of these probabilities should add to 1?

a) 1, 2
b) 3, 4
c) Both (a) and (b).
d) Neither (a) nor (b).

Estimating probabilities with training data

- To estimate $P(\text{spam})$, we compute

$$P(\text{spam}) \approx \frac{\text{\# spam emails in training set}}{\text{\# emails in training set}}$$

- To estimate $P(\text{ham})$, we compute

$$P(\text{ham}) \approx \frac{\text{\# ham emails in training set}}{\text{\# emails in training set}}$$

- What about $P(\text{features} \mid \text{spam})$ and $P(\text{features} \mid \text{ham})$?

Assumption of conditional independence

- Note that P(features | spam) looks like

$$P(x^{(1)} = 0, x^{(2)} = 1, \dots, x^{(d)} = 0 \mid \text{spam})$$

- Recall: the key assumption that the Naive Bayes classifier makes is that **the features are conditionally independent given the class**.

- This means we can estimate P(features | spam) as

$$\begin{aligned} &P(x^{(1)} = 0, x^{(2)} = 1, \dots, x^{(d)} = 0 \mid \text{spam}) \\ =&P(x^{(1)} = 0 \mid \text{spam}) \cdot P(x^{(2)} = 1 \mid \text{spam}) \cdot \dots \cdot P(x^{(d)} = 0 \mid \text{spam}) \end{aligned}$$

Concrete example

- Dictionary: “prince”, “money”, “free”, and “just”.

- Dataset of 5 emails (red are spam, green are ham):
  - **“I am the prince of UCSD and I demand money.”**
  - **“Tapioca Express: redeem your free Thai Iced Tea!”**
  - **“DSC 10: free points if you fill out CAPEs!”**
  - **“Click here to make a tax-free donation to the IRS.”**
  - **“Free career night at Prince Street Community Center.”**

Concrete example

- New email to classify: “Download a free copy of the Prince of Persia.”‘

Uh oh...

- What happens if we try to classify the email "just what's your price, prince"?

Smoothing

- **Without** smoothing:

$$P(x^{(i)} = 1 \mid \text{spam}) \approx \frac{\#\text{ spam containing word } i}{\#\text{ spam containing word } i + \#\text{ spam not containing word } i}$$

- **With** smoothing:

$$P(x^{(i)} = 1 \mid \text{spam}) \approx \frac{(\#\text{ spam containing word } i) + 1}{(\#\text{ spam containing word } i) + 1 + (\#\text{ spam not containing word } i) + 1}$$

- When smoothing, we add 1 to the count of every group whenever we're estimating a conditional probability.

Concrete example with smoothing

- What happens if we try to classify the email “just what’s your price, prince”?

Modifications and extensions

- **Idea:** Use pairs (or longer sequences) of words rather than individual words as features.
  - This better captures the dependencies between words.
  - It also leads to a much larger space of features, increasing the complexity of the algorithm.

Modifications and extensions

- **Idea:** Use pairs (or longer sequences) of words rather than individual words as features.
  - This better captures the dependencies between words.
  - It also leads to a much larger space of features, increasing the complexity of the algorithm.
- **Idea:** Instead of recording whether each word appears, record how many times each word appears.
  - This better captures the importance of repeated words.

Summary

Summary, next module

- Smoothing gives a way to make better predictions when a feature has never been encountered in the training data.
- The Naive Bayes classifier can be used for text classification, using the bag-of-words model.
- **Next module:** measuring performance of classifiers using precision and recall.