DSC 40A - Probability Roadmap

A hopefully helpful guide to solving probability questions

1. What is the sample space, S (all possible outcomes)? Consider using:

- individual objects
- sequences of objects
- sets of objects
- sequences of positions
- sets of positions

2. Are all of the outcomes in S equally likely, or are some more likely than others?

If all equally likely (most common), can use

$$P(\text{favorable outcome}) = \frac{\text{number of favorable outcomes in } S}{\text{total number of outcomes in } S}.$$

Then our probability question turns into two counting questions.

If not all equally likely, add up probabilities associated with each of the favorable outcomes, using

$$P(E) = \sum_{s \text{ in } E} P(s).$$

3. What are the favorable outcomes in S? Try to write down one favorable outcome, then think about what was forced and what you were free to select in generating this example. This can help you count the number of favorable outcomes.

4. Is it helpful to use the complement rule? Are there many ways in which the favorable outcome can be achieved, and only a few ways in which it's not achieved? If so, try using

$$P(A) = 1 - P(\overline{A}).$$

The words "at least" often indicate to use the complement rule.

5. Is it helpful to use the multiplication rule? Do you need several things to happen all at once?

The multiplication rule says you can make the first thing happen, then you only have to worry about the second thing happening in the case that the first already happened.

$$P(A \cap B) = P(A) * P(B|A)$$

We can also use it in the other order:

$$P(A \cap B) = P(B) * P(A|B)$$

We can also use it for three or more events, by just making sure that the next thing happens, assuming that all prior things already happened. For example, for three events, the multiplication rule becomes

$$P(A \cap B \cap C) = P(A) * P(B|A) * P(C|(A \cap B)).$$

6. Do you need to calculate a conditional probability, the probability of an event happening when you know another has occurred? The words "given that", "assuming", and "if you know" are often hints that you're being asked to find a conditional probability.

The formula for conditional probability is just the same as the multiplication rule, written in a slightly different way:

$$P(A|B) = \frac{P(A \cap B)}{P(B)}.$$

When computing a conditional probability, you can either:

1. use the above formula, or
2. calculate the conditional probability directly.

For most problems, calculating the conditional probability directly is usually easier because using the formula requires you to find two probabilities, one for the numerator and one for the denominator, while a direct calculation is just one probability. Also, finding the probability of the intersection of two events can be harder than just finding the conditional probability.

When calculating a conditional probability directly, use the given information (the condition) to restrict the possible set of outcomes.

7. Is it helpful to use the addition rule? Can you break down the favorable outcomes into a reasonable number of cases? Do the cases overlap? Can any one outcome fall into multiple cases? If so, don't forget to subtract off the intersection to avoid counting these outcomes multiple times.

The basic addition rule for two cases is

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

If the cases don't overlap, the last term is zero and can be left out.

For more than two cases, if the cases don't overlap, you can simply add up the individual properties of each case. If they do overlap, the formula gets complicated very quickly. You only need to know up to three cases:

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(C \cap B) + P(A \cap B \cap C).$$

8. If you're counting sequences, can the sequences be organized according to templates? Templates are like cases, showing a pattern that the sequence will follow. Templates should be disjoint, not overlapping.

If each template is associated with the same probability, then each term in the addition rule has the same value so we can calculate the probability as:

$$(\text{number of templates}) * (\text{probability of each template}).$$

9. Can you think of an additional assumption that would make the probability easier to calculate? Can you calculate how likely that assumption is to be true? If so, you may be able to break up your probability into cases based on this extra assumption and whether it is true or not.

This approach basically uses the Law of Total Probability. If A is the event you want the probability of, and B is the extra assumption, then since B and $\overline{B}$ partition the sample space, the Law of Total Probability says

$$\begin{aligned} P(A) &= P(A \cap B) + P(A \cap \overline{B}) \\ &= P(A|B) * P(B) + P(A|\overline{B}) * P(\overline{B}) \end{aligned}$$