DSC 40A - Group Work Session 6

due Monday, May 20th at 11:59PM

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. **One person** from each group should submit your solutions to Gradescope and **tag all group members** so everyone gets credit.

This worksheet won't be graded on correctness, but rather on good-faith effort. Even if you don't solve any of the problems, you should include some explanation of what you thought about and discussed, so that you can get credit for spending time on the assignment.

In order to receive full credit, you must work in a group of two to four students for at least 50 minutes in your assigned discussion section. You can also self-organize a group and meet outside of discussion section for 80 percent credit. You may not do the groupwork alone.

1 Probability

Most probability questions can be solved by applying one of the basic probability rules in the right way. Sometimes, some cleverness is needed to define the right sample space or the right events. There are often many ways to solve the same problem, some easier than others. It's really useful to learn multiple ways of doing the same problem, which will help you develop your problem-solving skills.

Here are the basic probability rules you'll need to use to solve the questions that follow.

Addition Rule:

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

Multiplication Rule:

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B|A)$$

Complement Rule:

$$\mathbb{P}(\overline{A}) = 1 - \mathbb{P}(A)$$

Conditional Probability:

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)}$$

Problem 1.

You're listening to a YouTube playlist of the 14 songs in Oscar Peterson's album, *Solo*. Suppose you're listening on shuffle, and each time a new song starts, it's equally likely to be any of the 14 songs on the playlist, regardless of which songs have been played so far. How many songs must you listen to so that the probability of hearing "Mirage" is at least 75%?

Problem 2.

Suppose we scramble the 26 letters of the alphabet in a random order so that each rearrangement is equally likely. What is the probability that the letters ABC wind up next to each other in that order?

2 Combinatorics

In probability, when all outcomes in the sample space are equally likely, the probability of an event is the number of outcomes in the event divided by the number of outcomes in the sample space. Thus, the probability of the event reduces to two counting (or combinatorics) questions, which ask *how many* outcomes are possible. When solving a counting question, it helps to write down one example outcome, then try to think about how many options we had at each step of generating this example.

There are a few basic combinatorial objects that we've studied in this class, namely sequences, permutations, and combinations. The hard part is often determining which one to use in which situation, which comes down to two important questions:

- Does the order in which I select the objects matter? In other words, does it count as different or the same if I choose the same objects in a different order?
- Am I selecting objects with or without replacement? In other words, am I allowed to have repeated objects?

Sequences:

A sequence is obtained by selecting k elements from a group of n possible elements with replacement, such that order matters. The number of sequences is

$$n^k.$$

Permutations:

A permutation is obtained by selecting k elements from a group of n possible elements without replacement, such that order matters. The number of permutations is

$$P(n,k) = \frac{n!}{(n-k)!}.$$

Combinations:

A combination is obtained by selecting k elements from a group of n possible elements without replacement, such that order does not matter. The number of combinations is

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{(n-k)!k!}.$$

Problem 3. Herb Garden

You want to plant an herb garden, so you go to a garden store that has 50 different herbs: 28 are culinary herbs, 12 are medicinal herbs, and 10 are aromatic herbs. You select 5 herbs for your herb garden by taking a random sample **without replacement** from the 50 available herbs.

a) If you consider the herbs you select as a combination (i.e. the order in which you select each herb does not matter), how many combinations of 5 herbs are possible?

b) If you consider the herbs you select as a combination (i.e. the order in which you select each herb does not matter), how many combinations of 5 herbs include 2 culinary herbs and 3 aromatic herbs?

c) If you consider the herbs you select as a permutation (i.e. the order in which you select each herb matters), how many permutations of 5 herbs are possible?

d) If you consider the herbs you select as a permutation (i.e. the order in which you select each herb matters), how many permutations of 5 herbs include 2 culinary herbs and 3 aromatic herbs?

e) What is the probability that you choose 2 culinary herbs and 3 aromatic herbs for your garden?

Problem 4. Shuffling Strings

a) How many different strings can be created by shuffling the letters of DOG?

b) How many different strings can be created by shuffling the letters of GAG?

Hint: The answer is not 6.

c) How many different strings can be created by shuffling the letters of GAAAGGGG?

Hint: How can you use combinations?

d) How many different strings can be created by shuffling the letters of AGGRAVATE?