Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. One person from each group should submit your solutions to Gradescope and tag all group members so everyone gets credit.

This worksheet won't be graded on correctness, but rather on good-faith effort. Even if you don't solve any of the problems, you should include some explanation of what you thought about and discussed, so that you can get credit for spending time on the assignment.

In order to receive full credit, you must work in a group of two to four students for at least 50 minutes in your assigned discussion section. You can also self-organize a group and meet outside of discussion section for 80 percent credit. You may not do the groupwork alone.

## 1 Naive Bayes

Naive Bayes is a classification algorithm that uses Bayes' Theorem to predict what category an object belongs to, based on certain features. In this section, we'll work out an example of Naive Bayes by hand.

## Problem 1.

In parts of the world other than San Diego, the weather changes from day to day. In these places, people try to guess tomorrow's weather using the current conditions.

Weather data for 20 random days in Columbus, Ohio are recorded below, along with the next day's weather (rainy, cloudy, or sunny).

| Next Day's Weather | Humidity | Temperature | Air Pressure |
| :---: | ---: | ---: | ---: |
| Rainy | $>50 \%$ | Cool | Low |
| Rainy | $>50 \%$ | Hot | Low |
| Rainy | $>50 \%$ | Cool | Low |
| Rainy | $25 \%-50 \%$ | Hot | High |
| Rainy | $25 \%-50 \%$ | Hot | Low |
| Rainy | $25 \%-50 \%$ | Cool | Low |
| Rainy | $25 \%-50 \%$ | Cool | Low |
| Rainy | $<25 \%$ | Cool | Low |
| Rainy | $<25 \%$ | Hot | Low |
| Rainy | $<25 \%$ | Hot | High |
| Cloudy | $>50 \%$ | Cool | Low |
| Cloudy | $>50 \%$ | Cool | Low |
| Cloudy | $25 \%-50 \%$ | Hot | High |
| Cloudy | $<25 \%$ | Cool | High |
| Cloudy | $<25 \%$ | Cool | Low |
| Sunny | $>50 \%$ | Cool | Low |
| Sunny | $>50 \%$ | Hot | High |
| Sunny | $>50 \%$ | Cool | High |
| Sunny | $25 \%-50 \%$ | Hot | High |
| Sunny | $<25 \%$ | Hot | High |
|  |  |  |  |

a) Suppose that today's humidity is $>50 \%$, the temperature is hot, and the air pressure is low. Use Naive Bayes without smoothing to predict whether tomorrow will be rainy, cloudy, or sunny. Show your work.
b) Repeat the algorithm with smoothing to predict whether tomorrow will be rainy, cloudy, or sunny given that today's humidity is $>50 \%$, the temperature is hot, and the air pressure is low. Show your work.

## 2 Roulette Game

## Problem 2.

Consider a roulette wheel that has 36 numbers colored red (R) or black (B) according to the following pattern:

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| R | R | R | R | R | B | B | B | B | R | R | R | R | B | B | B | B | B |
| 36 | 35 | 34 | 33 | 32 | 31 | 30 | 29 | 28 | 27 | 26 | 25 | 24 | 23 | 22 | 21 | 20 | 19 |

For example, this means the color of number 2 is red and the color of number 28 is black.
Define the following three events:

- Let $A$ be the event that a spin of the wheel yields a red number,

$$
A=\{1,2,3,4,5,10,11,12,13,24,25,26,27,32,33,34,35,36\}
$$

- Let $B$ be the event that a spin of the wheel yields an even number,

$$
B=\{2,4,6,8,10,12,14,16,18,20,22,24,26,28,30,32,34,36\}
$$

- Let $C$ be the event that a spin of the wheel yields a number no greater than 18 ,

$$
C=\{1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16,17,18\}
$$

a) Are the events $A, B$, and $C$ pairwise independent? That is, is event $A$ independent of event $B$, event $A$ independent of event $C$, and event $B$ independent of event $C$ ?
b) Are the events $A$ and $B$ conditionally independent conditioned on $C$ ?

