
Mock Final Exam 2 - DSC 40A, Spring 2024

Full Name:

PID:

Seat Number:

Instructions:

- This mock exam consists of 13 questions. **Advice: Read all of the questions before starting to work, because the questions are not sorted by difficulty.**
- You have 60 minutes to complete this mock exam.
- **Note that the main purpose of this mock exam is to give you practice with answering unfamiliar questions in an exam-like environment. The questions on the real exam may be formatted differently than those here.**
 - Specifically, you may have some long-answer proof-style questions (similar to homework questions), though there aren't any such questions here.
 - Most of the questions here are themed around a common topic, though your exam may not have a theme.

By signing below, you are agreeing that you will behave honestly and fairly during and after this exam.

Signature:

Background

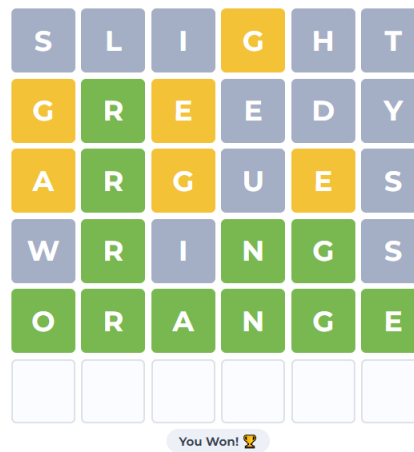
In the game Stringle, players try to guess a randomly generated string. There is a new Stringle string available each day.

Each day's Stringle string is a **six-letter string**, where each letter is chosen uniformly at random, **with replacement**, from among the 26 letters of the English alphabet. This means Stringle strings can have repeated letters, and they do not need to have any meaning as an English word.

Players input a guess, which can be any six-letter string, and get feedback on their guess, which is given by coloring the letters of the guessed string as follows:

- **green** for letters in the correct position,
- **yellow** for letters in the Stringle string, but in the incorrect position,
- **gray** for letters not in the Stringle string.

Based on this feedback, they can guess again, an **unlimited** number of times, until they guess the string correctly, at which point the game is over. Here's an example of what happens when someone plays Stringle and guesses the string on their fifth guess:



A player's Stringle **score** is the number of guesses it took them to guess the word correctly. In the example above, the player's score is 5.

Question 1

Stringle uses the following color scheme, or assignment of colors to meanings:

- **green** for letters in the correct position,
- **yellow** for letters in the Stringle string, but in the incorrect position,
- **gray** for letters not in the Stringle string.

Suppose the game developers could have instead used any three colors from the set of seven colors {red, orange, yellow, green, blue, purple, gray}, so long as no color has multiple meanings. How many color schemes include the color purple?

- 15
- 30
- 90
- 120
- None of the above.

Question 2

Suppose that every day, the Stringle string is chosen without replacement from among the set of all possible valid strings. What is the probability that DSCTEN was the Stringle string for one of the first 100 days after the game's release?

Question 3

What is the probability of today's Stringle string having exactly one duplicate letter, like SYSTEM, BEFORE, or YELLOW?

- $\frac{P(26, 5) \cdot 5!}{26^6}$
- $\frac{\binom{26}{5} \cdot 5!}{26^6}$
- $\frac{\binom{6}{2} \cdot P(26, 5)}{26^6}$
- $\frac{\binom{26}{5}}{26^6}$
- None of the above.

Question 4

What is the probability of today's Stringle string having at least one duplicate letter, like SYSTEM, GOOGLE, or WAHOOO?

Question 5

What is the probability that today's String string has the last three letters matching the first three letters, like BAMBAM or GOOGOO, if we know that the first letter matches the fourth letter?

- $\frac{1}{26^2}$
- $\frac{1}{26^3}$
- $\frac{P(26, 3)}{P(26, 5)}$
- $\frac{P(26, 3)}{26^6}$
- None of the above.

Question 6

How many possible Stringle strings contain exactly three H's, like HAHAHA or SSSHHH?

- $\binom{6}{3} \cdot 26^3$
- $\binom{6}{3} \cdot 26^6$
- $\binom{6}{3}$
- 25^3
- None of the above.

Question 7

For this problem, we'll say that there are six vowels: A, E, I, O, U, and Y. Consider the following three events:

- A is the event that today's Stringle string starts with a vowel.
- B is the event that today's Stringle string starts with a letter in the first half of the alphabet (A through M, inclusive).
- C is the event that today's Stringle string does not start with a Z.

Which of the following is true?

- A and B are independent. A and B are conditionally independent given C .
- A and B are independent. A and B are conditionally dependent given C .
- A and B are dependent. A and B are conditionally independent given C .
- A and B are dependent. A and B are conditionally dependent given C .
- None of the above.

Question 8

For this problem, we'll say that there are six vowels: A, E, I, O, U, and Y. What is the probability that today's Stringle string satisfies at least one of the following conditions?

- starts with a vowel
- ends with a vowel
- first two letters are the same

Question 9

Your first guess for today's Stringle has six distinct letters. There are 3^6 possible colored feedback sequences for this first guess, since each of the six letters could be colored one of three colors: green, yellow, or gray. How many of the 3^6 possible feedback sequences for your first guess include 2 green letters, 2 yellow letters, and 2 gray letters?

- $6!$
- $\binom{6}{3}$
- $\left(\binom{6}{2}\right)^3$
- $\binom{6}{2} \cdot \binom{4}{2}$
- None of the above.

Question 10

Consider the following three events:

- A is the event that today's Stringle string has a duplicate letter.
- B is the event that today's Stringle string starts with a B .
- C is the event that today's Stringle string has the same first and last letter.

Which of the following is true?

- A and B are independent.
 A and C are independent.
 B and C are dependent.
- A and B are dependent.
 A and C are independent.
 B and C are independent.
- A and B are independent.
 A and C are dependent.
 B and C are dependent.
- A and B are dependent.
 A and C are dependent.
 B and C are independent.
- None of the above.

Question 11

Stringle has become such a popular game that many spin-offs have been created. In a variant called Quadringle, players have to solve four different Stringle puzzles at the same time. We'll say that a Quadringle puzzle is determined by which four six-letter strings it includes, and the order of those four strings does not matter. How many Quadringle puzzles are possible?

- $(26^6)^4$
- $\binom{26^6}{4}$
- $P(26^6, 4)$
- $4^{C(26,6)}$
- None of the above.

Question 12

In today's Quadringle puzzle, the four strings are all very similar:

DEALER, RELAYS, ALLOYS, and SALADS.

Suppose you pick one of these four strings at random, so that each has a $\frac{1}{4}$ probability of being chosen. Then you choose one letter of that word at random, such that each has a $\frac{1}{6}$ probability of being chosen.

It turns out that you picked an L. What is the probability that your L came from the word SALADS?

- $\frac{1}{24}$
- $\frac{1}{6}$
- $\frac{1}{5}$
- $\frac{1}{4}$
- None of the above.

Question 13

Consider the function $R(h)$:

$$R(h) = \sqrt{(h-3)^2 + 1} = ((h-3)^2 + 1)^{\frac{1}{2}}$$

$R(h)$ is a convex, differentiable function with only one local minimum.

- a) Perform two iterations of gradient descent, using an initial guess of $h_0 = 2$ and a learning rate of $\alpha = 2\sqrt{2}$. Show your work, and your final answers, h_1 and h_2 .

- b) With more iterations, will we eventually converge to the minimizer? Why or why not?