DSC 40A - Group Work Session 4
due Wednesday, August 28th at 11:59PM

Write your solutions to the following problems by either typing them up or handwriting them on another piece of paper. **One person** from each group should submit your solutions to Gradescope and **tag all group members** so everyone gets credit.

This worksheet won't be graded on correctness, but rather on good-faith effort. Even if you don't solve any of the problems, you should include some explanation of what you thought about and discussed, so that you can get credit for spending time on the assignment.

In order to receive full credit, you must work in a group of two to four students for at least 50 minutes in your assigned discussion section. You can also self-organize a group and meet outside of discussion section for 80 percent credit. You may not do the groupwork alone.

1 Probability

Most probability questions can be solved by applying one of the basic probability rules in the right way. Sometimes, some cleverness is needed to define the right sample space or the right events. There are often many ways to solve the same problem, some easier than others. It's really useful to learn multiple ways of doing the same problem, which will help you develop your problem-solving skills.

Here are the basic probability rules you'll need to use to solve the questions that follow.

Addition Rule:

$$\mathbb{P}(A \cup B) = \mathbb{P}(A) + \mathbb{P}(B) - \mathbb{P}(A \cap B)$$

Multiplication Rule:

$$\mathbb{P}(A \cap B) = \mathbb{P}(A) \cdot \mathbb{P}(B|A)$$

Complement Rule:

$$\mathbb{P}(\overline{A}) = 1 - \mathbb{P}(A)$$

Conditional Probability:

$$\mathbb{P}(B|A) = \frac{\mathbb{P}(A \cap B)}{\mathbb{P}(A)}$$

Problem 1.

A bitstring is a sequence of 0s and 1s. For example, 0110100 is a bitstring of length 7.

Suppose that we generate a bitstring of length 4 such that each digit is equally likely to be a 0 or 1.

a) What is the probability that the bitstring is 1111?

> **Solution:** Write your solution here.

b) What's the probability that the bitstring contains at least one 0 and one 1?

Solution: Write your solution here.

c) What is the probability that a bitstring has more 0s than 1s?

Solution: Write your solution here.

d) What is the probability that a bitstring has more 0s than 1s, if we know that the first bit is a 0?

Solution: Write your solution here.

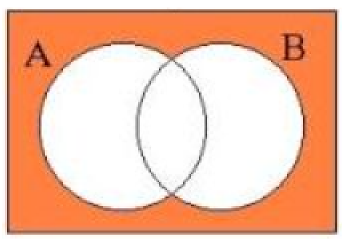
e) Suppose now that you generate two bitstrings and look at one of them. You see that this bitstring has more 0s than 1s. What is the probability that in total, for both strings together, there are more 0s than 1s?

Solution: Write your solution here.

Problem 2.

Let A and B be two independent events in the sample space S. Show that $\bar{A}$ and $\bar{B}$ must be independent of one another.

You may use the fact that $P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$, which should be apparent from the Venn diagram below.



Solution: Write your solution here.

Problem 3.

Suppose you have 6 pairs of socks in your sock drawer, each in a different pattern. It is still dark out in the morning when you get dressed, so you randomly pull one sock at a time out of the drawer, until you have removed two matching socks. What is the probability that you pull out exactly 5 socks from your sock drawer in the morning?

Solution: Write your solution here.

Problem 4.

You're listening to a YouTube playlist of the 14 songs in Oscar Peterson's album, *Solo*. Suppose you're listening on shuffle, and each time a new song starts, it's equally likely to be any of the 14 songs on the playlist, regardless of which songs have been played so far. How many songs must you listen to so that the probability of hearing "Mirage" is at least 75%?

Solution: Write your solution here.

Problem 5.

Suppose we scramble the 26 letters of the alphabet in a random order so that each rearrangement is equally likely. What is the probability that the letters ABC wind up next to each other in that order?

Solution: Write your solution here.

2 Combinatorics

In probability, when all outcomes in the sample space are equally likely, the probability of an event is the number of outcomes in the event divided by the number of outcomes in the sample space. Thus, the probability of the event reduces to two counting (or combinatorics) questions, which ask *how many* outcomes are possible. When solving a counting question, it helps to write down one example outcome, then try to think about how many options we had at each step of generating this example.

There are a few basic combinatorial objects that we've studied in this class, namely sequences, permutations, and combinations. The hard part is often determining which one to use in which situation, which comes down to two important questions:

- Does the order in which I select the objects matter? In other words, does it count as different or the same if I choose the same objects in a different order?
- Am I selecting objects with or without replacement? In other words, am I allowed to have repeated objects?

Sequences:

A sequence is obtained by selecting k elements from a group of n possible elements with replacement, such that order matters. The number of sequences is

$$n^k.$$

Permutations:

A permutation is obtained by selecting k elements from a group of n possible elements without replacement, such that order matters. The number of permutations is

$$P(n,k) = \frac{n!}{(n-k)!}.$$

Combinations:

A combination is obtained by selecting k elements from a group of n possible elements without replacement, such that order does not matter. The number of combinations is

$$C(n,k) = \binom{n}{k} = \frac{P(n,k)}{k!} = \frac{n!}{(n-k)!k!}.$$

Problem 6. Herb Garden

You want to plant an herb garden, so you go to a garden store that has 50 different herbs: 28 are culinary herbs, 12 are medicinal herbs, and 10 are aromatic herbs. You select 5 herbs for your herb garden by taking a random sample **without replacement** from the 50 available herbs.

a) If you consider the herbs you select as a combination (i.e. the order in which you select each herb does not matter), how many combinations of 5 herbs are possible?

Solution: Write your solution here.

b) If you consider the herbs you select as a combination (i.e. the order in which you select each herb does not matter), how many combinations of 5 herbs include 2 culinary herbs and 3 aromatic herbs?

Solution: Write your solution here.

c) If you consider the herbs you select as a permutation (i.e. the order in which you select each herb matters), how many permutations of 5 herbs are possible?

Solution: Write your solution here.

d) If you consider the herbs you select as a permutation (i.e. the order in which you select each herb matters), how many permutations of 5 herbs include 2 culinary herbs and 3 aromatic herbs?

Solution: Write your solution here.

e) What is the probability that you choose 2 culinary herbs and 3 aromatic herbs for your garden?

Solution: Write your solution here.

Problem 7. Shuffling Strings

a) How many different strings can be created by shuffling the letters of DOG?

Solution: Write your solution here.

b) How many different strings can be created by shuffling the letters of GAG?

Hint: The answer is not 6.

Solution: Write your solution here.

c) How many different strings can be created by shuffling the letters of GAAAGGGG?

Hint: How can you use combinations?

Solution: Write your solution here.

d) How many different strings can be created by shuffling the letters of AGGRAVATE?

Solution: Write your solution here.

Problem 8. Xs and Os

Let $N(a,b)$ represent the number of strings you can create out of a Xs and b Os. Explain why $N(a,b)$ satisfies each of the following:

$$N(0,b) = 1 \tag{1}$$
$$N(a,0) = 1 \tag{2}$$
$$N(a,b) = N(a-1,b) + N(a,b-1) \quad \text{for } a > 0 \text{ and } b > 0. \tag{3}$$

Solution: Write your solution here.

Problem 9. Tiebreaker

To break a tie among a group of $n \geq 3$ people, you come up with the following tiebreaker: Everyone flips a coin. If one person's coin is different from all the others, that person wins, and the tie is broken! Otherwise, repeat the process.

a) What is the probability that the tie is broken after the first coin toss?

Solution: Write your solution here.

b) Fix an integer $k \geq 1$. Find the probability that the tie is broken after exactly k coin tosses?

Solution: Write your solution here.