# Midterm 2 - DSC 40A, Winter 2024

**Instructions**

- This is a 50-minute exam consisting of 4 questions worth a total of 40 points.
- The only allowed resource is your hand-written reference notes.
- No calculators.
- Please write neatly and stay within the provided boxes.
- You may fill out the **front page only** until you are instructed to start.

**Statement of Academic Integrity**

By submitting your exam, you are attesting to the following statement of academic integrity.

*I will act with honesty and integrity during this exam.*

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| Name: | Solutions |
| PID: | A12345678 |
| Seat you are in: | |

## Version - A

**1.** (6 points) Quarks is one of the smallest fundamental particles in physics. There are 6 types of quarks: up, down, top, bottom, strange, charm. Each one of the 6 quarks can be in two state: quark or antiquark.

**a)** Consider an experiment where we select n quarks uniformly at random. The **result** of the experiment is a description of the **type and state** of the quark selected. For example, if $n = 3$, one possible result is:

- Selected quark 1 is a top quark.
- Selected quark 2 is a charm antiquark.
- Selected quark 3 is a top antiquark.

How many results are possible for this experiment with $n$ quarks?

○ $6^n$

○ $9^n$

● $12^n$

○ $18^n$

○ $36^n$

○ None of the above.

**b)** A meson is formed by combining two quarks. In order to form a meson, the two quarks must satisfy the following rules:

- They must be in **different state**: one must be quark and the other one must be anti-quark
- The two quarks can be the **same type**: i.e. top quark and top antiquark can form a meson
- The **order** of quark and antiquark does matter.

Consider an experiment where we select n mesons uniformly at random. How many results are possible for this experiment?

○ $2^n$

○ $6^n$

○ $12^n$

○ $18^n$

● $36^n$

○ None of the above.

**2.** (12 points) A special poker card deck contains the 52 standard card:

Heart: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A
Diamond: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A
Club: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A
Spade: 2, 3, 4, 5, 6, 7, 8, 9, 10, J, Q, K, A

Plus two wildcard: Red Joker and Black Joker. The total numbers of card in this card deck is 54.

**a)** (2 points) Ho many ways to select a 4 card hand from this card deck? (Note: order does not matter in a card hand.)

$\frac{54!}{4!50!}$ or $C(54, 4)$

**b)** (4 points) For this deck, how many 5 card hands are there that include a four-of-a-kind (four cards of the same value)? Show your work.

**Proof:**

**Solution:** Since the first 4 cards are four-of-a-kind (same number), there are 13 ways to select four-of-a-kind. For the 5th card, there are 50 total choices (12 values $\times$ 4 suits $\times$ + 2 wildcards). So there are total 13*50=650 options.

**c)** (6 points) In certain poker rules, a bomb is defined as either four-of-a-kind, or two wildcards (red joker and black joker). Suppose you randomly draw 4 card hand and you found a bomb in it, what is the probability that the bomb is four-of-a-kind? Show your work.

**Proof:**

**Solution:** The number of sequences that containing two jokers (the other two cards are arbitrary) is given by:

$$\underbrace{C(4,2)}_{\text{Joker Locations}} \times P(2,2) \times C(52,2) \times P(2,2) = 4 \times 3 \times 52 \times 51$$

The number of sequences that form four of a kind is

$$C(13,1)P(4,4) = 13 \times 4 \times 3 \times 2$$

The two events are exclusive. And for a bomb to occur, the four cards either contains two jokers or form four of a kind. Hence,

$$P(\text{Four of a kind}|\text{Bomb}) = \frac{13 \times 4 \times 3 \times 2}{13 \times 4 \times 3 \times 2 + 4 \times 3 \times 52 \times 51} = \frac{1}{103}$$

**3.** (16 points) Schrödinger's cat is a famous thought experiment in quantum mechanics proposed by physicist Erwin Schrödinger in 1935. The experiment is described below:

> "Imagine there's a hypothetical cat in a closed box with a toxic radioactive element that might decay. If it decays, the cat dies; if it doesn't, the cat lives."

Please note that Schrödinger's Cat is a purely theoretical concept—a thought experiment. It has never been executed in the real world, and no cats have ever been harmed as a result of it.

**a)** (6 points) Suppose the cat has 90% of the chance to die if the decay happens. The cat also has 10% chance to die even if the decay does not happen. Suppose the decay happens with a 20% probability. After you open the box, you find the cat dead. What is the probability that the decay happened?

**Proof:**

**Solution:** The probability we are looking for is $P(Decay|Dead)$. Using Baye's equation, we can write:

$$P(Decay|Dead) = \frac{P(Dead|Decay)P(Decay)}{P(Dead)}$$

Using the Law of Total Probability, we can rewrite the denominator:

$$P(Decay|Dead) = \frac{P(Dead|Decay)P(Decay)}{P(Dead|Decay)P(Decay) + P(Dead|NotDecay)P(NotDecay)}$$

We know that:

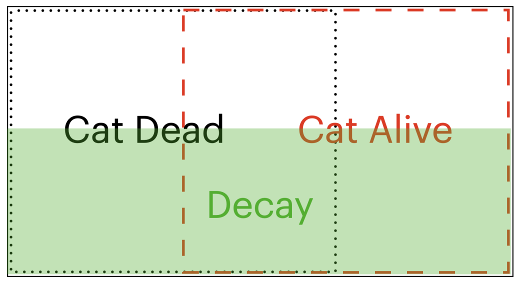
$$\begin{aligned} P(Dead|Decay) &= 90\% = 0.9 \\ P(Dead|NotDecay) &= 10\% = 0.1 \\ P(Decay) &= 20\% = 0.2 \end{aligned}$$

Plugging these numbers, we have:

$$\begin{aligned} P(Decay|Dead) &= \frac{0.9 \times 0.2}{0.9 \times 0.2 + 0.1 \times (1 - 0.2)} \\ &= \frac{18}{26} \end{aligned}$$

Quantum superposition is a mind-bending and counter-intuitive concept in physics. In Schrödinger's cat scenario, when the box is closed, the cat can be **both dead and alive** simultaneously. However, once we open the box and observe, the superposition collapses, and the cat must be either dead or alive, not both.

**b)** (4 points) The Venn Diagram below depicts Schrödinger's cat scenario with the box closed. In this diagram, the black dotted line is the event of cat dead, the red dashed line denotes the event of cat being alive, and the shaded region is the event of decay.




Based on this Venn diagram, which of the following is true? Select all that applies:

■ Decay is independent of Cat Dead

■ Decay is independent of Cat Alive

☐ Cat Dead and Cat Alive are mutually exclusive.

☐ Cat Dead, Cat Alive, and Decay form a partition of the sample space.

☐ None of Above.

**c)** (6 points) Given the following probabilities:

- P(Cat Dead $\cup$ Decay) $= \frac{4}{5}$
- P(Cat Alive $\cup$ Decay) $= \frac{1}{2}$
- P(Cat Alive $\cup$ Cat Dead) $= 1$
- P(Decay) $= \frac{1}{5}$

Using the Venn diagram in part b), calculate the probability for Schrodinger's cat to be in superposition state (i.e. both dead and alive):

**Proof:**

**Solution:** Since we know that the decay is independent to cat's state, we have

$$\begin{aligned} P(Dead \cup Decay) &= P(Dead) + P(Decay) - P(Dead \cap Decay) \\ &= P(Dead) + P(Decay) - P(Dead) \cdot P(Decay) = \frac{4}{5} \end{aligned}$$

Similarly, we have:

$$\begin{aligned} P(Alive \cup Decay) &= P(Alive) + P(Decay) - P(Alive \cap Decay) \\ &= P(Alive) + P(Decay) - P(Alive) \cdot P(Decay) = \frac{1}{2} \end{aligned}$$

Plugging in P(Decay) = $\frac{1}{5}$, we have:

$$P(Dead) + \frac{1}{5} - \frac{1}{5}P(Dead) = \frac{4}{5}$$
$$P(Alive) + \frac{1}{5} - \frac{1}{5}P(Alive) = \frac{1}{2}$$

Solving these two equations, we have:

$$P(Dead) = \frac{3}{4}$$
$$P(Alive) = \frac{3}{8}$$

The probability of superposition state is P(Alive ∩ Dead). Since we know that P(Alive ∪ Dead) = 1 we have:

$$P(Alive \cup Dead) = P(Alive) + P(Dead) - P(Alive \cap Dead)$$

Rearranging terms, we have:

$$\begin{aligned} P(Alive \cap Dead) &= P(Alive) + P(Dead) - P(Alive \cup Dead) \\ &= \frac{3}{4} + \frac{3}{8} - 1 \\ &= \frac{6}{8} + \frac{3}{8} - 1 \\ &= \frac{1}{8} \end{aligned}$$

**4.** (6pt) The standard acceleration of gravity $g$ is a universal constant of nature. That means $g = 9.80665$ across the entire universe. It is also known that:

$$G = m \times g \tag{1}$$

Where $G$ is the weight of an object and m is the mass of an object. Issac is interested in measuring g. He prepared a dumbbell with mass 10 in his garage and measured the weight of it 3 times. The measured weight is 102, 98, 100

**a)** (1pt) Calculate the mean value of $g$ Issac measured.

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**b)** (2pt) After his measurement, an alien from the Frequentist Galaxy visited Issac's garage. Alien from Frequentist Galaxy could only understand frequentist statistics. Explain to the alien why Issac's measurement deviates from $g = 9.80665$ (Hint: verbal explanation is enough, no equation is needed):

**Explanation:**

**Solution:** Frequentist does not assign probability to hypothesis, so g is always equal to 9.80665 and it does not change. Issac's measurement deviate from the actual g value because of uncertainties in the measurement.

**c)** (2pt) After the first alien left, another alien from the Bayesian galaxy arrived. This alien told Issac: "I think g could take any value: it could be 1, could be 10, could be 100.". Using the Bayes equation and Issac's measurement, convince the alien that g should be close to 10.

**Explanation:**

**Solution:** The Bayes equation reads:

$$Posterior = \frac{Prior \times Likelihood}{Evidence}$$

The alien provides a uniform prior, or P(Hypothesis). They assume g could take many different possible values. However, the likelihood term is P(Data—Hypothesis). For the hypothesis of g = 1, since it does not agree with data, the likelihood is very low and thereby does not contribute to posterior; the same argument hold for g = 100. Only when g=10 will the likelihood be high.

**d)** (1pt) Which explanation do you prefer? (Note: both answers are correct, enjoy your one free point).

◯ Frequentist ◯ Bayesian

**Solution:** Both are correct.