

Lecture 17 - Sequences, Permutations, and Combinations



DSC 40A, Winter 2024

Announcements

- ▶ Homework 5 is due **Tonight**.
 - ▶ Please come to office hours if you have questions!
- ▶ **Important:** We've posted **many** probability resources on the [resources tab of the course website](#). These will no doubt come in handy.
 - ▶ No more DSC 40A-specific readings, though the Probability Roadmap was written specifically for students of this course.



HDSI Undergraduate Social

All HDSI Undergrads and Faculty Welcome

Friday February 23rd

3-5pm

HDSL Patio

Join us for light refreshments and
meet fellow students and faculty
before finals start!

DDS AND HDSI DEI COMMITTEE JOINT EVENT

Agenda

- ▶ Sequences, permutations, and combinations.

Sequences, permutations, and combinations

Motivation

- ▶ Many problems in probability involve counting.
 - ▶ Suppose I flip a fair coin 100 times. What's the probability I see 34 heads?
 - ▶ Suppose I draw 3 cards from a 52 card deck. What's the probability they all are all from the same suit?
- ▶ In order to solve such problems, we first need to learn how to count.
- ▶ The area of math that deals with counting is called **combinatorics**.

Selecting elements (i.e. sampling)

- ▶ Many experiments involve choosing k elements randomly from a group of n possible elements. This group is called a **population**.
 - ▶ If drawing cards from a deck, the population is the deck of all cards.
 - ▶ If selecting people from DSC 40A, the population is everyone in DSC 40A.
- ▶ Two decisions:
 - ▶ Do we select elements with or without **replacement**?
 - ▶ Does the **order** in which things are selected matter?

Sequences

- ▶ A **sequence** of length k is obtained by selecting k elements from a group of n possible elements **with replacement**, such that **order matters**.
- ▶ **Example:** Draw a card (from a standard 52-card deck), put it back in the deck, and repeat 4 times.
- ▶ **Example:** A UCSD PID starts with “A” then has 8 digits. How many UCSD PIDs are possible?

Sequences

In general, the number of ways to select k elements from a group of n possible elements such that **repetition is allowed** and **order matters** is n^k .

(Note: We mentioned this fact in the lecture on clustering!)

Permutations

- ▶ A **permutation** is obtained by selecting k elements from a group of n possible elements **without replacement**, such that **order matters**.

- ▶ **Example:** Draw 4 cards (without replacement) from a standard 52-card deck.

- ▶ **Example:** How many ways are there to select a president, vice president, and secretary from a group of 8 people?

Permutations

- ▶ In general, the number of ways to select k elements from a group of n possible elements such that **repetition is not allowed** and **order matters** is

$$P(n, k) = (n)(n - 1)\dots(n - k + 1)$$

- ▶ To simplify: recall that the definition of $n!$ is

$$n! = (n)(n - 1)\dots(2)(1)$$

- ▶ Given this, we can write

$$P(n, k) = \frac{n!}{(n - k)!}$$

Discussion Question

UCSD has 7 colleges. How many ways can I rank my top 3 choices?

- a) 21
- b) 210
- c) 343
- d) 2187
- e) None of the above.

Special case of permutations

- ▶ Suppose we have n people. The total number of ways I can rearrange these n people in a line is

- ▶ This is consistent with the formula

$$P(n, n) = \frac{n!}{(n - n)!} = \frac{n!}{0!} = \frac{n!}{1} = n!$$

Combinations

- ▶ A **combination** is a set of k items selected from a group of n possible elements **without replacement**, such that **order does not matter**.
- ▶ **Example:** There are 24 ice cream flavors. How many ways can you pick two different flavors?

From permutations to combinations

- ▶ There is a close connection between:
 - ▶ the number of permutations of k elements selected from a group of n , and
 - ▶ the number of combinations of k elements selected from a group of n

$$\# \text{ combinations} = \frac{\# \text{ permutations}}{\# \text{ orderings of } k \text{ items}}$$

- ▶ Since $\# \text{ permutations} = \frac{n!}{(n-k)!}$ and $\# \text{ orderings of } k \text{ items} = k!$, we have

$$C(n, k) = \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

Combinations

In general, the number of ways to select k elements from a group of n elements such that **repetition is not allowed** and **order does not matter** is

$$\binom{n}{k} = \frac{n!}{(n-k)!k!}$$

The symbol $\binom{n}{k}$ is pronounced “ n choose k ”, and is also known as the **binomial coefficient**.

Example: committees

- ▶ How many ways are there to select a president, vice president, and secretary from a group of 8 people?

- ▶ How many ways are there to select a committee of 3 people from a group of 8 people?

- ▶ If you're ever confused about the difference between permutations and combinations, **come back to this example.**

Discussion Question

A domino consists of two faces, each with anywhere between 0 and 6 dots. A set of dominoes consists of every possible combination of dots on each face.

How many dominoes are in the set of dominoes?

- a) $\binom{7}{2}$
- b) $\binom{7}{1} + \binom{7}{2}$
- c) $P(7, 2)$
- d) $\frac{P(7,2)}{P(7,1)} 7!$

Summary

Summary

- ▶ A **sequence** is obtained by selecting k elements from a group of n possible elements with replacement, such that order matters.
 - ▶ Number of sequences: n^k .
- ▶ A **permutation** is obtained by selecting k elements from a group of n possible elements without replacement, such that order matters.
 - ▶ Number of permutations: $P(n, k) = \frac{n!}{(n-k)!}$.
- ▶ A **combination** is obtained by selecting k elements from a group of n possible elements without replacement, such that order does not matter.
 - ▶ Number of combinations: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.