Lecture 18 - Probabability and Combinatorics Examples

DSC 40A, Winter 2024

Announcements

- Homework 6 is posted and due next Wednesday.
- HDSI undergrad & faculty mixer will be this afternoon 3-5pm at HDSI patio
  - Light refreshment will be provided

Agenda

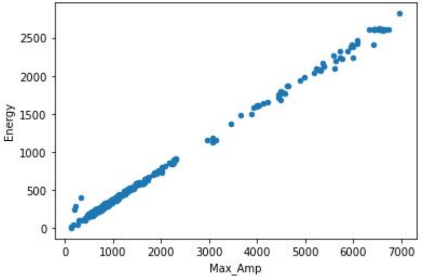
- Invited Algorithm Presentation
- Review of combinatorics.
- Lots of examples.

Invited Algorithm Presentation: Owen Shi

HW4 Algorithm

Owen Shi

In [5]:
```
waveforms.plot(kind='scatter', x='Max_Amp', y='Energy');
```

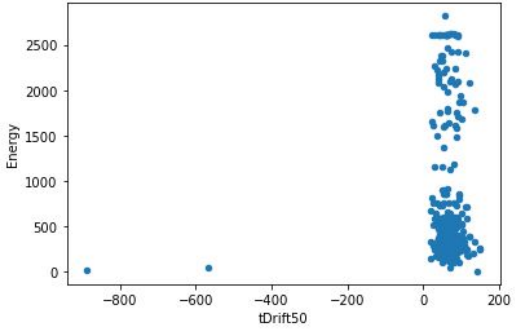


In [4]:
```
waveforms = pd.read_csv('HPGeData.csv')
waveforms.head()
```

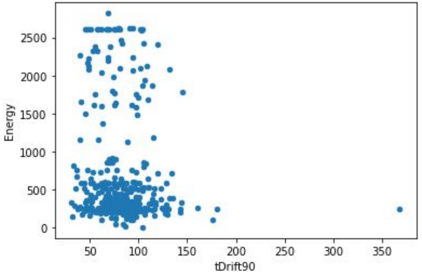
Out[4]:

	Max_Amp	tDrift50	tDrift90	tDrift100	blnoise	tslope
0	1233.0	61.0	69.0	81.0	11.5110	-108.1349
1	1319.0	93.0	116.0	135.0	3.7505	-112.9078
2	1237.0	81.0	89.0	104.0	2.3472	-111.2230
3	4469.0	90.0	98.0	112.0	1.8966	-111.4219
4	796.0	84.0	95.0	117.0	2.9256	-123.4542

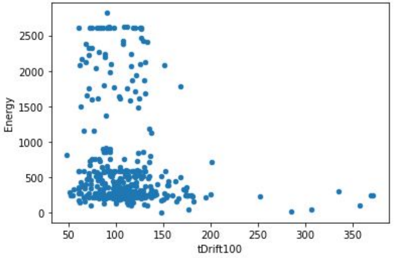
In [6]:
```
waveforms.plot(kind='scatter', x='tDrift50', y='Energy');
```



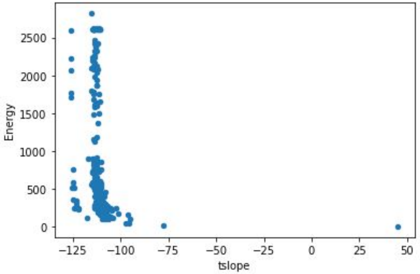
In [8]:
```
waveforms.plot(kind='scatter', x='tDrift90', y='Energy');
```



In [9]:
```
waveforms.plot(kind='scatter', x='tDrift100', y='Energy');
```



In [11]:
```
waveforms.plot(kind='scatter', x='tslope', y='Energy');
```



The Framework

```
In [5]: def design_matrix(d):
            df = d.copy()
            df['Intercept'] = 1
            feat1 = (1 / df['blnoise']**2 + 1 / df['tslope']) / df['Max_Amp']
            feat2 = (1 / df['blnoise'])**1.5 / (df['Max_Amp'] + df['blnoise']**3)
            feat3 = (df['Max_Amp'] * df['blnoise'] * df['tslope']) / \
                    (df['Max_Amp']**2 + 1 / df['blnoise'] + 1 / df['tslope'])
            df['feat1'] = feat1
            df['feat2'] = feat2
            df['feat3'] = feat3
        #     df.plot(kind='scatter', y='Energy', x='feat3')
            return df.get(['Intercept', 'Max_Amp', 'feat1', 'feat2', 'feat3']).to_numpy()

        design_matrix(waveforms)
```

```
In [6]: def observation_vector():
            return waveforms['Energy']
```

```
In [31]: np.random.seed(10)
         lambdas = np.logspace(-10, 10, 100)
         mses = []
         for l in lambdas:
             mse = 0
             w = w_star(l)
             for i in range(waveforms.shape[0]):
                 row = waveforms.iloc[i]
                 feat1 = 1 / row[4] + 1 / row[5]
                 feat2 = 1 / row[5] * 1 / feat1
                 X = pd.Series([1, row[0], feat1, feat2])
                 pred = X @ w
                 mse += (pred - row['Energy']) ** 2
             mse /= waveforms.shape[0]
             mses.append(mse)

         lambdas[np.argmin(mses)]

Out[31]: 4.132012400115335e-09
```

```
In [ ]: best_lambda = 0
```

```
[7]: def w_star(lam):
         X = design_matrix(waveforms)
         y = observation_vector()
         return np.linalg.inv(X.T @ X + lam * np.eye(X.shape[1])) @ X.T @ y
```

```python
def design_matrix(d):
    df = d.copy()
    df['Intercept'] = 1
    return df.get(['Intercept', 'Max_Amp']).to_numpy()
```

```python
def design_matrix(d):
    df = d.copy()
    df['Intercept'] = 1
    feat1 = 1 / df['blnoise']
    feat2 = 1 / df['tslope']
    df['feat1'] = feat1
    df['feat2'] = feat2
    return df.get(['Intercept', 'Max_Amp', 'feat1', 'feat2']).to_numpy()
```

```python
def design_matrix(d):
    df = d.copy()
    df['Intercept'] = 1
    feat1 = 1 / df['blnoise'] + 1 / df['tslope']
    feat2 = 1 / df['tslope'] * 1 / (1 / df['blnoise'] + 1 / df['tslope'])
    feat3 = df['tslope'] / df['Max_Amp']
    df['feat1'] = feat1
    df['feat2'] = feat2
    df['feat3'] = feat3
    return df.get(['Intercept', 'Max_Amp', 'feat1', 'feat2', 'feat3']).to_numpy()
```

```python
def design_matrix(d):
    df = d.copy()
    df['Intercept'] = 1
    feat1 = (1 / df['blnoise'] + 1 / df['tslope']) / df['Max_Amp']
    feat2 = df['Max_Amp'] / df['blnoise']
    feat3 = df['Max_Amp'] / df['tslope']
    df['feat1'] = feat1
    df['feat2'] = feat2
    df['feat3'] = feat3
    return df.get(['Intercept', 'Max_Amp', 'feat1', 'feat2', 'feat3']).to_numpy()
```

Submission History

#	Submitted On (PST)	Submitters	Score	Active
87	Feb 16 at 11:44 PM	OS	9.0	
86	Feb 16 at 11:43 PM	OS	0.0	
85	Feb 16 at 11:40 PM	OS	9.0	
84	Feb 16 at 10:01 PM	OS	9.0	
83	**Feb 16 at 9:58 PM**	OS	**9.0**	✔

```
without_f1 = 793.7921119778865
without_f2 = 766.2772193373274
without_f3 = 727.4278430354087
```

In [5]:
```
def design_matrix(d):
    df = d.copy()
    df['Intercept'] = 1
    feat1 = (1 / df['blnoise']**2 + 1 / df['tslope']) / df['Max_Amp']
    feat2 = (1 / df['blnoise'])**1.5 / (df['Max_Amp'] + df['blnoise']**3)
    feat3 = (df['Max_Amp'] * df['blnoise'] * df['tslope']) / \
            (df['Max_Amp']**2 + 1 / df['blnoise'] + 1 / df['tslope'])
    df['feat1'] = feat1
    df['feat2'] = feat2
    df['feat3'] = feat3
#     df.plot(kind='scatter', y='Energy', x='feat3')
    return df.get(['Intercept', 'Max_Amp', 'feat1', 'feat2', 'feat3']).to_numpy()

design_matrix(waveforms)
```

One More Extra Credit Opportunity

- Building a Naive Bayes classifier to separate neutrino signals from unwanted noises!
  - This one will be **Optional:** chances to earn extra credit, but does not count as part of homework problem.
  - Will be released and due together wit HW7
  - More details in the following weeks.
- The full HPGe dataset is released at `https://zenodo.org/records/8257027`
  - In raw waveform format, no extracted parameters.

Extra Credit Rules

- The classifier competition will earn you up to 10% extra credit on Midterm 2, depending on your leaderboard ranking
  - Same as the energy regression challenge
- However, the maximum extra credit you can earn from both challenges is capped at 10%
- Example: Owen ranked No. 2 on regression challenge, he will get 9% EC on Midterm 1, so the maximum amount of EC he can get on Midterm 2 is 1%
- This is to encourage students who did not get EC from the regression challenge to participate.

Review of combinatorics

Combinatorics as a tool for probability

- If S is a sample space consisting of equally-likely outcomes, and A is an event, then $P(A) = \frac{|A|}{|S|}$.
- In many examples, this will boil down to using permutations and/or combinations to count $|A|$ and $|S|$.
- **Tip:** Before starting a probability problem, always think about what the sample space S is!

Sequences

- A **sequence** of length k is obtained by selecting k elements from a group of n possible elements **with replacement**, such that **order matters**.

- **Example:** You roll a die 10 times. How many different sequences of results are possible?

Sequences

In general, the number of ways to select k elements from a group of n possible elements such that **repetition is allowed** and **order matters** is

$$n^k.$$

Permutations

- A **permutation** is obtained by selecting k elements from a group of n possible elements **without replacement**, such that **order matters**.
- **Example:** How many ways are there to select a president, vice president, and secretary from a group of 8 people?

Permutations

- In general, the number of ways to select k elements from a group of n possible elements such that **repetition is not allowed** and **order matters** is

$$
\begin{aligned}
P(n,k) &= (n)(n-1)\ldots(n-k+1) \\
&= \frac{n!}{(n-k)!}
\end{aligned}
$$

Combinations

- A **combination** is a set of k items selected from a group of n possible elements **without replacement**, such that **order does not matter**.

- **Example:** How many ways are there to select a committee of 3 people from a group of 8 people?

Combinations

In general, the number of ways to select k elements from a group of n elements such that **repetition is not allowed** and **order does not matter** is

$$\begin{aligned} C(n,k) &= \binom{n}{k} \\ &= \frac{P(n,k)}{k!} \\ &= \frac{n!}{(n-k)!k!} \end{aligned}$$

The symbol $\binom{n}{k}$ is pronounced "n choose k", and is also known as the **binomial coefficient**.

Lots of examples

Discussion Question

A domino consists of two faces, each with anywhere between 0 and 6 dots. A set of dominoes consists of every possible combination of dots on each face.

How many dominoes are in the set of dominoes?

a) $\binom{7}{2}$

b) $\binom{7}{1} + \binom{7}{2}$

c) $P(7, 2)$

d) $\frac{P(7,2)}{P(7,1)} 7!$

Selecting students — overview

We're going answer the same question using several different techniques.

Question 1: There are 20 students in a class. Avi is one of them. Suppose we select 5 students in the class uniformly at random **without replacement**. What is the probability that Avi is among the 5 selected students?

Selecting students (Method 1: using permutations)

Question 1: There are 20 students in a class. Avi is one of them. Suppose we select 5 students in the class uniformly at random **without replacement**. What is the probability that Avi is among the 5 selected students?

Selecting students (Method 2: using permutations and the complement)

Question 1: There are 20 students in a class. Avi is one of them. Suppose we select 5 students in the class uniformly at random **without replacement**. What is the probability that Avi is among the 5 selected students?

Selecting students (Method 3: using combinations)

Question 1: There are 20 students in a class. Avi is one of them. Suppose we select 5 students in the class uniformly at random **without replacement**. What is the probability that Avi is among the 5 selected students?

Selecting students (Method 3: using combinations)

Question 1, Part 1 (Denominator): If you draw a sample of size 5 at random without replacement from a population of size 20, how many different **sets** of individuals could you draw?

Selecting students (Method 3: using combinations)

Question 1, Part 2 (Numerator): If you draw a sample of size 5 at random without replacement from a population of size 20, how many different **sets** of individuals include Avi?

Selecting students (Method 3: using combinations)

Question 1: There are 20 students in a class. Avi is one of them. Suppose we select 5 students in the class uniformly at random **without replacement**. What is the probability that Avi is among the 5 selected students?

Selecting students (Method 4: "the easy way")

Question 1: There are 20 students in a class. Avi is one of them. Suppose we select 5 students in the class uniformly at random **without replacement**. What is the probability that Avi is among the 5 selected students?

With vs. without replacement

Discussion Question

We've determined that a probability that a random sample of 5 students from a class of 20 **without replacement** contains Avi (one student in particular) is $\frac{1}{4}$.

Suppose we instead sampled **with replacement**. Would the resulting probability be equal to, greater than, or less than $\frac{1}{4}$?

a) Equal to
b) Greater than
c) Less than

Summary

Summary

- A **sequence** is obtained by selecting k elements from a group of n possible elements with replacement, such that order matters.
  - Number of sequences: n^k.
- A **permutation** is obtained by selecting k elements from a group of n possible elements without replacement, such that order matters.
  - Number of permutations: $P(n,k) = \frac{n!}{(n-k)!}$.
- A **combination** is obtained by selecting k elements from a group of n possible elements without replacement, such that order does not matter.
  - Number of combinations: $\binom{n}{k} = \frac{n!}{(n-k)!k!}$.