# DSC 40A: Theoretical Foundations of Data Science

Lecture 13 Part II
**Feature engineering and data transformations**

October 27, 2025

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- Content: Lectures 1-13, Homeworks 1-4, Groupworks 1-5

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- Prepare by practicing old exam problems on practice.dsc40a.com

# Announcements

Varun and Owen will be hosting a midterm review session this Thursday 10/30 from 5pm-7pm in **Ledden Auditorium** (near HSS/APM)

Stay tuned for further details via Campuswire

# Recap from last week

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**Question:** What do each of these have in common?

These are **all linear** in the weights $w_i$.

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**Example** A nonlinear hypothesis function

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This function is **nonlinear** in both the weights and the feature $x$. We can create a new hypothesis function $T(x) = b_0 + b_1 x$, which is linear in the weights $b_0, b_1$, by applying the transformation

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The weights are related by the equations $b_0 = \ln(w_0)$ and $b_1 = w_1$.

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We will explore this in an interactive notebook, continuing from last week's example.

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You would like to model $y_i$, the liters of finished bottled product during week $i$, in terms of measurable quantities.

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$$H(\vec{x}) = w_0 (x^{(1)})^{w_1} (x^{(2)})^{w_2} (x^{(3)})^{w_3}.$$

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where we have defined the transformed features

$$z^{(j)} = \ln(x^{(j)}), \quad j = 1, 2, 3,$$

using the normal equations to obtain optimal weights $b_0^*, b_1^*, b_2^*, b_3^*$.

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Finally, we can recover the optimal weights for the original hypothesis via

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\begin{aligned}
w_0^* &= e^{b_0^*}, \\
w_1^* &= b_1^*, \\
w_2^* &= b_2^*, \\
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Note: unlike before, we need to transform our *features* as well as our *weights*!

# Question

Answer at **q.dsc40a.com**.

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Which of the following hypothesis functions is **not** linear in the parameters?

(A) $H(\vec{x}) = w_1 \left(x^{(1)} x^{(2)}\right) + \frac{w_2}{x^{(1)}} \sin\left(x^{(2)}\right)$

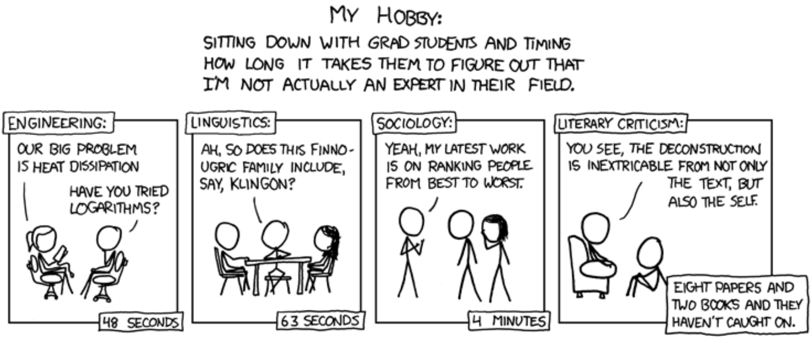
(B) $H(\vec{x}) = 2^{\,w_1}\, x^{(1)}$

(C) $H(\vec{x}) = \vec{w} \cdot \text{Aug}(\vec{x})$

(D) $H(\vec{x}) = w_1 \cos\left(x^{(1)}\right) + w_2\, 2^{\,x^{(2)} \log x^{(3)}}$

(E) More than one of the above.

# Have you tried using logarithms?




**xkcd #451**

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Hypothesis functions that are linear in the parameters are much easier to work with.

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- After the Midterm Exam, we'll switch gears to **probability theory**.

# DSC 40A: Theoretical Foundations of Data Science

Lecture 14 Part I
**Gradient Descent**

October 27, 2025

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$$\frac{df}{dt}(t) = 5t^4 - t^3 - 5t^2 + 2t - 9$$

- Then what?

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