DSC 40B - Discussion 02

Problem 1.

Consider the code below where heights is an array of n elements:

```
for i in range(n):
        for j in range(2*i):
                height = heights[i] + heights[j]
```

What is the time complexity of the code?

Problem 2.

Are the following asymptotic bounds true?

1. $n \log n = O(n^2)$
2. $\left(\frac{1}{n}\right)^3 = O(\frac{1}{n})^4$
3. $n^{\frac{1}{2}} = \Omega(n^{\frac{1}{3}})$
4. $e^n = O(n!)$
5. $\log n = \Omega(\sqrt{n})$

Problem 3.

a) Let $f(n) = 12log_2(3^{n^2-2n} + 2^{logn} - 10n^2 - log_3 n)$. Which of the following asymptotic bounds on f is true?

1. $f(n) = \Omega(1)$
2. $f(n) = O(1)$
3. $f(n) = O(\log n)$
4. $f(n) = \Omega(\log n)$
5. $f(n) = \Theta(\log n)$
6. $f(n) = O(n)$
7. $f(n) = O(e^n)$
8. $f(n) = \Theta(n)$
9. $f(n) = \Theta(n^2)$

b) What is the best case time complexity of the following function?

```
def foo(arr):
''' arr is a sorted array of size n'''
        i = 0
        j = len(arr) - 1

        while i < j:
                current_sum = arr[i] + arr[j]

                if current_sum == 5:
                        return sum(arr)
                elif current_sum < 5:
                        i += 1
                else:
                        j -= 1

        return False
```

Problem 4.

Consider the algorithm below.

```
def bogosearch(numbers, target):
    """search by randomly guessing. `numbers` is an array of n numbers"""
    n = len(numbers)

    while True:
        # randomly choose a number between 0 and n-1 in constant time
        guess = np.random.randint(n)
        if numbers[guess] == target:
            return guess
```

We will set up the analysis of the expected time complexity of this algorithm.

a) What are the cases? How many are there?

b) What is the probability of case α?

c) What is the running time in case α?

Problem 5.

Provide a tight theoretical lower bound for the problems given below. Provide justification for your answer.

a) Given an array of n numbers, find the sum of the numbers in the array.

b) Given a sorted array of $n \geq 2$ numbers, find the second largest number in the array.