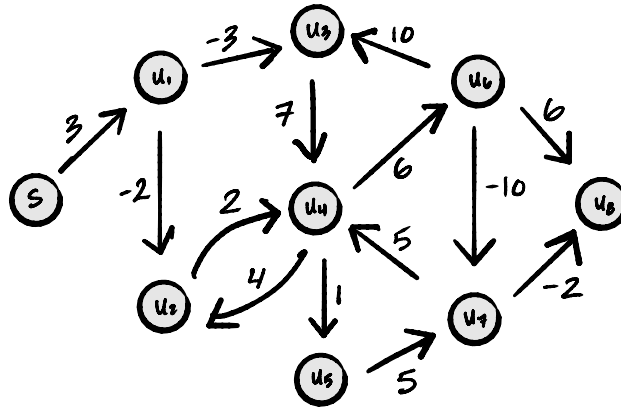# DSC 40B - Homework 08

Due: Sunday, November 30

Write your solutions to the following problems by handwriting them on another piece of paper. Unless otherwise noted by the problem's instructions, show your work or provide some justification for your answer. Homeworks are due via Gradescope at 11:59 p.m.

Problem 1.

Run Bellman-Ford on the following graph using node s as the source. Below each node u, write the shortest path length from s to u. Mark the predecessor of u by highlighting it or making a bold arrow. You can assume that `graph.edges` produces the graph's edges in the following order:

$$(u_2, u_4), (u_7, u_8), (u_6, u_3), (s, u_1), (u_1, u_2), (u_4, u_5), (u_1, u_3),$$
$$(u_3, u_4), (u_6, u_7), (u_4, u_6), (u_5, u_7), (u_7, u_4), (u_4, u_2), (u_6, u_8)$$
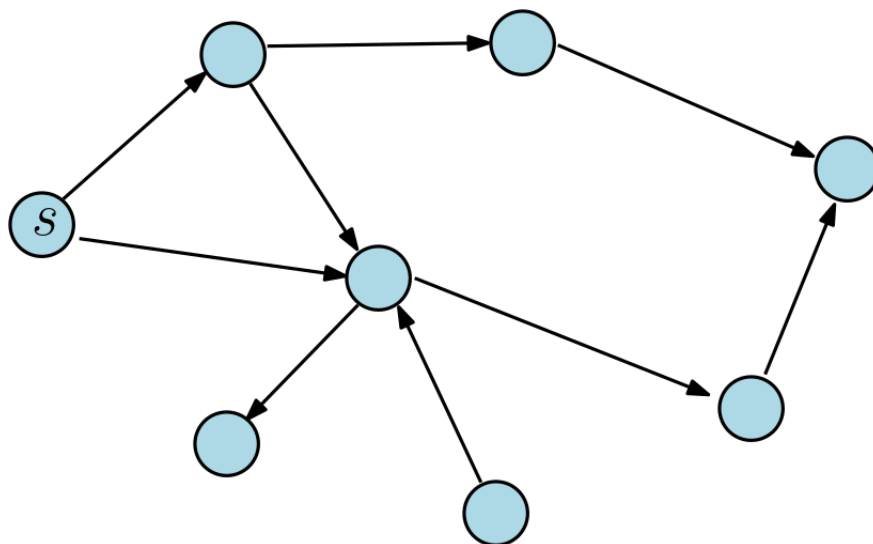

Problem 2.

Recall that the Bellman-Ford algorithm (with early stopping) will terminate early if, after updating every edge, no predecessors have changed. Suppose it is known that the greatest shortest path distance in a graph $G = (V, E)$ has $\Theta(\sqrt{V})$ edges. What is the worst case time complexity of Bellman-Ford when run on this graph? State your answer using Θ notation.
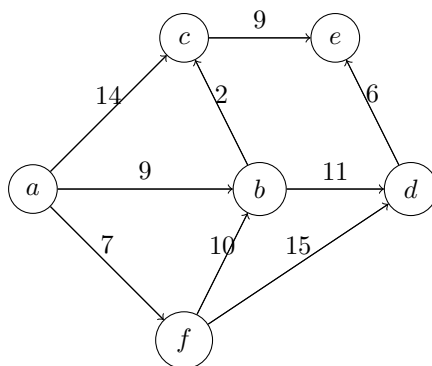
Problem 3.

Suppose we are given a directed and weighted graph $G = (V, E)$ that look like the following figure. It is known that there are no negative cycles.

Suppose the Bellman-Ford algorithm **with early stopping** is run on this graph shown above using node s as the source (the edge weights are purposely not shown). In the worst case, how many iterations of the outer for-loop of Bellman-Ford will be performed? Briefly justify your answer.

Problem 4.

Run Dijkstra's Algorithm on the following graph using node a as the source. Below each node u, write the shortest path length from a to u. Mark the predecessor of u by highlighting it or making a bold arrow.

Problem 5.

True or False. Suppose $G = (V, E, \omega)$ is a weighted graph for which all edges are positive, except for those edges of a node s which may or may not be negative. If Dijkstra's algorithm is run on G with s as the source, the correct shortest paths will be found. Assume that the graph does not have any negative loops. Justify your answer.

Problem 6.

Suppose $G = (V, E, \omega)$ is a weighted graph without negative cycles. Suppose we have an edge $(u, v) \in E$ whose weight is $w(u, v) = 5$. Let $s \in V$ be the source node. Let $\delta(s, u)$ and $\delta(s, v)$ denote the shortest path distance from s to u and to v, respectively. For each statement below, if it is true, provide an example to illustrate it; otherwise, explain why it is false.

a) (True or False): Now suppose that G is a **directed** graph. Then, it is possible that $\delta(s, u) = 50$ but $\delta(s, v) = 10$.

b) (True or False): Now suppose that G is a **undirected** graph. Then, it is possible that $\delta(s, u) = 50$ but $\delta(s, v) = 10$.